

# Nonparametric intensity estimation from indirect point process observations under unknown error distribution

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## Synopsis

We consider the nonparametric estimation of the intensity function of a Poisson point process in a circular model from indirect observations  $N_1, \dots, N_n$ . These observations emerge from hidden point process realizations with the target intensity through contamination with additive error. Under the assumption that the error distribution is unknown and only available by means of an additional sample  $Y_1, \dots, Y_m$  we derive minimax rates of convergence with respect to the sample sizes  $n$  and  $m$  under abstract smoothness conditions and propose an orthonormal series estimator which attains the optimal rate of convergence. The performance of the estimator depends on the correct specification of a dimension parameter whose optimal choice relies on smoothness characteristics of both the intensity and the error density. Since a priori knowledge of such characteristics is a too strong assumption, we propose a data-driven choice of the dimension parameter based on model selection and show that the adaptive estimator either attains the minimax optimal rate or is suboptimal only by a logarithmic factor.

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## 1. Introduction

Point process models are used in a wide variety of applications, including, amongst others, stochastic geometry [Sto+13], extreme value theory [Res87], and queueing theory [Bré81]. Each realization of a point process is a random set of points  $\{x_j\}$  which can alternatively be represented as an  $\mathbb{N}_0$ -valued random measure  $\sum_j \delta_{x_j}$  where  $\delta_\bullet$  denotes the Dirac measure concentrated at  $\bullet$ . Poisson point processes (PPPs) are of particular importance since they serve as the elementary building blocks for more complex point process models. Let  $\mathbb{X}$  be a locally compact second countable Hausdorff space,  $\mathcal{X}$  the corresponding Borel  $\sigma$ -field and  $\Lambda$  a locally finite measure on the measurable space  $(\mathbb{X}, \mathcal{X})$ , i.e.,  $\Lambda(C) < \infty$  for all relatively compact sets  $C$  in  $\mathcal{X}$ . A random set of points  $N = \{x_j\}$  from  $\mathbb{X}$  (resp. the random measure  $N = \sum_j \delta_{x_j}$ ) is called *Poisson point process* with *intensity measure*  $\Lambda$  if

- the number  $N_C = |N \cap C|$  of points located in  $C$  follows a Poisson distribution with parameter  $\Lambda(C)$  for all relatively compact  $C \in \mathcal{X}$ , and

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- for all  $n \in \mathbb{N}$  and disjoint sets  $A_1, \dots, A_n \in \mathcal{X}$ , the random variables  $N_{A_1}, \dots, N_{A_n}$  are independent.

It is well-known that the distribution of a PPP is completely determined by its intensity measure. Hence, from a statistical point of view, the (non-parametric) estimation of the intensity measure or its Radon-Nikodym derivative (the *intensity function*) with respect to some dominating measure from observations of the point process is of fundamental importance.

Inference and testing problems for Poisson and more general point processes have been tackled in a wide range of scenarios. The monographs [Kar91] and [Kut98] offer a comprehensive overview and discuss both parametric and nonparametric methods. From a methodological point of view, our approach in this paper is related to the article [RB03] where the estimation of the intensity function from direct observations was considered. As in the present paper, the analysis of the adaptive estimator in [RB03] is based on appropriate concentration inequalities for Poisson point processes. Whereas the concentration inequalities developed and applied in [RB03] represent analogues of results for random variables due to [Mas00], one main tool of our approach are analogues of concentration inequalities due to [KR05] that we have derived in a separate manuscript [Kro16]. Other approaches to nonparametric intensity estimation from direct observations, without making a claim to be exhaustive, can be found in [BB09] (where the performance of a histogram estimator under Hellinger loss is analysed), [Bir07] (using a testing approach to model selection), [GN00] (using a minimum complexity estimator in the Aalen model), and [PW04] (suggesting a wavelet estimator in the multiplicative intensity model).

Theoretical work on intensity estimation has recently been motivated by applications to genomic data. The model considered in the article [Big+13] is motivated by data arising throughout the processing of DNA ChIP-seq data. The article [San14] takes its motivation from the analysis of genomic data as well. In that paper, the author studies nonparametric inference of a so-called *reproduction function* from one realization of an aggregated point process and additional observations related to the model. The observations in that paper are not of the same type as the ones that we will consider here, although the estimation problem can also be ascribed to the area of intensity estimation from indirect observations. In addition, let us mention two further articles where the development of nonparametric statistical methods for the analysis of point processes was inspired by applications from biology: first, motivated through DNA sequencing techniques, the article [SZ12] introduces a change-point model for nonhomogeneous Poisson processes occurring in molecular biology. Second, the article [ZK10] considered the nonparametric inference of Cox process data by means of a kernel type estimator.

Usually one aims to estimate the intensity function  $\lambda$  from *direct* observations  $\tilde{N}_1, \dots, \tilde{N}_n$  where

$$\tilde{N}_i = \sum_j \delta_{x_{ij}} \quad (1.1)$$

are realizations of a PPP with the target intensity  $\lambda$ .

In this paper, however, we assume that we are interested in the nonparametric estimation of the intensity function  $\lambda$  without having access to the observations in (1.1). Instead, we are in the setup of a *Poisson inverse problem* [AB06] where we can only observe  $N_1, \dots, N_n$  given through

$$N_i = \sum_j \delta_{y_{ij}}. \quad (1.2)$$

The *indirect* observations  $N_i$  are related to the hidden  $\tilde{N}_i$  by the identity  $y_{ij} = x_{ij} + \varepsilon_{ij} - \lfloor x_{ij} + \varepsilon_{ij} \rfloor$ . The definition of the  $y_{ij}$  as the fractional part of the additively contaminated  $x_{ij}$  yields a circular model by means of the usual topological identification of the interval  $[0, 1)$  and the circle of perimeter 1. Assumptions concerning the intensity and the errors will be discussed below in more detail.

Note that the circular definition of the general model (1.2) is convenient to model the case of periodic intensity functions which is of particular importance in applications: periodic intensity functions are suitable to model the occurrence of events that are subject to a natural temporal period (day, week,  $\dots$ ), for instance financial transactions or gun crimes. We refer the interested reader to [HMZ03] for references concerning the wide range of applications.

The model (1.2) is closely related to (circular) deconvolution models. In the circular deconvolution problem (CDP) one aims at estimating the density  $g$  of a random variable  $X$  with values in  $[0, 1)$  from repeated observations  $Y_i = X_i + \varepsilon_i - \lfloor X_i + \varepsilon_i \rfloor$ . Here,  $\varepsilon_i$  i.i.d.  $\sim f$  for some error density  $f$ . The CDP and its analogue extension on the real line (where  $X$  is real-valued and the observations are  $Y_i = X_i + \varepsilon_i$ ) have been treated in a wide range of research articles (cf. [Mei09] for a comprehensive introduction to the subject).

Note that, in contrast to our approach, the few existing papers on Poisson inverse problems ([CJ02], [AB06], [Big+13]) assume the error distribution to be known. This conservative assumption is also present in most of the research literature dealing with (circular) deconvolution problems. If the error density is unknown, even identifiability of the statistical model is not guaranteed. Thus, several remedies have been introduced to overcome this problem: for instance, it is possible to impose additional assumptions on the statistical model (cf., for instance, the article [SVB10] which deals with blind convolution under additive centred Gaussian noise with unknown variance). Alternatively, one can consider a framework with panel data [Neu07]. Finally, one can assume the availability of an additional sample from the error density (cf., for instance, [DH93], [Joh09], [CL10], [CL11]) to guarantee identifiability and enable inference. In this paper, we will stick to this last option.

Let us assume that the errors  $\varepsilon_{ij}$  contributing to the general model (1.2) are stationary in the sense that  $\varepsilon_{ij} \sim f$  for some *unknown* error density  $f$ . Under this assumption, two models are of particular interest:

1. the errors  $\varepsilon_{ij}$  are i.i.d.  $\sim f$  (this case will be referred to as the *Poisson model* throughout the paper),
2. the errors coincide, that is  $\varepsilon_{ij} \equiv \varepsilon_i \sim f$  for all  $j$  (this case will be referred to as the *Cox model*)

(the names *Poisson* resp. *Cox* model will be justified below). Note that in the classical (circular) deconvolution problem a differentiation between distinct models as in the point process case is not possible. We will study nonparametric estimation of the intensity function  $\lambda$  under the Poisson and the Cox model from observations

$$N_1, \dots, N_n \text{ i.i.d.} \quad \text{and} \quad Y_1, \dots, Y_m \text{ i.i.d.} \sim f \quad (1.3)$$

where the  $N_i$  are given as in (1.2). A natural aim here is to detect optimal rates of convergence in terms of the sample sizes  $n$  and  $m$  and to construct adaptive estimators attaining these rates.

It has to be remarked that the Cox model has already been studied intensively in [Big+13]: the authors of that paper exclusively consider the case that the error density  $f$  is known (making the  $Y$  sample in (1.3) needless) and ordinary smooth, that is, the Fourier coefficients of  $f$  are polynomially decreasing. Note that our investigation in this paper allows to consider error densities with faster rates of decay. Under the stated assumptions, the article [Big+13] provides a remarkable proof of a minimax lower bound over Besov spaces and suggests a hard-thresholding wavelet series estimator which automatically adapts to unknown smoothness and attains a rate which is optimal up to a logarithmic factor. Moreover the authors show (cf. Theorems 6.1 and 6.2 in [Big+13]) that it is impossible to construct a consistent estimator of the intensity on the basis of a preceding 'alignment step'.

From a methodological point of view, our approach is inspired by the one conducted in [JS13]. In contrast to that paper, the proof of the minimax lower bound in our setup has to deal with

the different nature of the observations. The key argument here is that the Hellinger distance between Poisson point processes can be bounded from above by the Hellinger distance between the corresponding intensity measures. We consider orthonormal series estimators of the form

$$\hat{\lambda}_k = \sum_{0 \leq |j| \leq k} \widehat{[\lambda]}_j \mathbf{e}_j \quad (1.4)$$

where  $\mathbf{e}_j(\cdot) = \exp(2\pi i j \cdot)$  and  $\widehat{[\lambda]}_j$  is an appropriate estimator of the Fourier coefficient  $[\lambda]_j$  corresponding to the basis function  $\mathbf{e}_j(\cdot)$  (see Section 2 for details). Of course, this estimator is motivated by the  $\mathbb{L}^2$ -convergent representation  $\lambda = \sum_{j \in \mathbb{Z}} [\lambda]_j \mathbf{e}_j$  for square-integrable  $\lambda$ . It will turn out that the performance of the estimator  $\hat{\lambda}_k$  crucially depends on the choice of the dimension parameter  $k$  and that its optimal value depends on smoothness characteristics of the intensity that are usually not available in practice. In order to choose  $k$  in a completely data-driven manner, we follow an approach based on model selection (cf., for instance, [BBM99] or [Com15]) and select the dimension parameter as the minimizer of a penalized contrast criterion,

$$\hat{k} := \operatorname{argmin}_{0 \leq k \leq K_{nm}} \{\Upsilon(\hat{\lambda}_k) + \text{PEN}_k\},$$

where  $K_{nm} \in \mathbb{N}_0$  (depending both on the samples sizes  $n, m \in \mathbb{N}$  and the random observations) determines the set of admissible models,  $\Upsilon$  is a contrast function and  $\text{PEN}_k$  a penalty term which will be specified in more detail below. Our choice of the penalty term is non-deterministic but random already in the partially adaptive case where one assumes only the smoothness of the intensity to be unknown but the smoothness of the error density to be known. For the theoretical analysis of the adaptive estimator we need Talagrand type concentration inequalities tailored to the framework with PPP observations which cannot be directly transferred from results applied in the usual density estimation or deconvolution frameworks (cf. Remark 2.3 in [Kro16]). As already mentioned above, these inequalities have already been derived in a separate manuscript [Kro16], and only the specific results needed for our application in this paper are stated in the appendix.

The article is organized as follows: in Section 2 we introduce our methodological approach and in Section 3 we consider our nonparametric estimation problems from a minimax point of view. Then, Section 4 considers adaptive estimation of the intensity for the Poisson model, whereas Section 5 deals with adaptive estimation for the Cox model.

## 2. Methodology

### 2.1. Notation

Throughout this work we will assume that the intensity  $\lambda$  and the density  $f$  belong to the space  $\mathbb{L}^2 := \mathbb{L}^2([0, 1], dx)$  of square-integrable functions on the interval  $[0, 1]$ . Let  $\{\mathbf{e}_j\}_{j \in \mathbb{Z}}$  be the *complex trigonometric basis* of  $\mathbb{L}^2$  given by  $\mathbf{e}_j(t) = \exp(2\pi i j t)$ . The Fourier coefficients of a function  $g \in \mathbb{L}^2$  are denoted as follows:

$$[g]_j = \int_0^1 g(t) \mathbf{e}_j(-t) dt.$$

For a strictly positive symmetric sequence  $\omega = (\omega_j)_{j \in \mathbb{Z}}$  we introduce the weighted norm  $\|\cdot\|_\omega$  defined via  $\|g\|_\omega^2 := \sum_{j \in \mathbb{Z}} \omega_j |[g]_j|^2$ . The corresponding scalar product is denoted with  $\langle g, h \rangle_\omega = \sum_{j \in \mathbb{Z}} \omega_j [g]_j \overline{[h]_j}$ . Throughout the paper, we use the notation  $a(n, m) \lesssim b(n, m)$  if  $a(n, m) \leq C \cdot b(n, m)$  for some numerical constant  $C$  independent of  $n$  and  $m$ .

## 2.2. The minimax point of view

We will evaluate the performance of an arbitrary estimator  $\tilde{\lambda}$  of  $\lambda$  by means of the mean integrated weighted squared loss  $\mathbb{E}[\|\tilde{\lambda} - \lambda\|_{\omega}^2]$ . We will take up the minimax point of view and consider the *maximum risk* defined by

$$\sup_{\lambda \in \Lambda} \sup_{f \in \mathcal{F}} \mathbb{E}[\|\tilde{\lambda} - \lambda\|_{\omega}^2] \quad (2.1)$$

where  $\Lambda$  and  $\mathcal{F}$  are classes of potential intensity functions  $\lambda$  and densities  $f$ , respectively. Note that in the case of a known error density  $f$  one would only consider the supremum over all  $\lambda \in \Lambda$  for this fixed error density. If one is able to show an upper risk bound which holds uniformly for all  $f$  belonging to some class  $\mathcal{F}$ , one obtains an upper bound for the maximum risk in (2.1). In order to show minimax lower bounds in terms of the sample size  $m$  in (1.3), it will turn out sufficient to reduce the class  $\mathcal{F}$  even to a subset of two elements (see the proof of Theorem 3.3).

The *minimax risk* is defined via

$$\inf_{\tilde{\lambda}} \sup_{\lambda \in \Lambda} \sup_{f \in \mathcal{F}} \mathbb{E}[\|\tilde{\lambda} - \lambda\|_{\omega}^2]$$

where the infimum is taken over all estimators  $\tilde{\lambda}$  of  $\lambda$ . An estimator  $\lambda^*$  is called *rate optimal* if

$$\sup_{\lambda \in \Lambda} \sup_{f \in \mathcal{F}} \mathbb{E}[\|\lambda^* - \lambda\|_{\omega}^2] \lesssim \inf_{\tilde{\lambda}} \sup_{\lambda \in \Lambda} \sup_{f \in \mathcal{F}} \mathbb{E}[\|\tilde{\lambda} - \lambda\|_{\omega}^2].$$

The classes  $\Lambda$  of intensity functions and  $\mathcal{F}$  of densities to be considered in this article will be specified in Section 3 below where we derive lower bounds on the minimax risk for these specific choices and prove that this lower bound is attained up to a numerical constant by a suitably defined orthonormal series estimator.

## 2.3. Sequence space representation for the Poisson model

The orthogonal series estimator that we will introduce in the next subsection is crucially motivated by a sequence space representation of the considered models. This representation forms the point of origin for the definition of the estimators  $[\hat{\lambda}]_j$  of the Fourier coefficients in (1.4). We address ourselves now to the derivation of this sequence space model.

Under the Poisson model, the observed point processes  $N_1, \dots, N_n$  in (1.3) are generated from independent Poisson point processes  $\tilde{N}_1, \dots, \tilde{N}_n$  with intensity function  $\lambda$  by independent random contaminations of the individual points. We emphasize again that the (unobserved) contaminations are assumed to follow a probability law given by an unknown density  $f$  and are to be understood additively modulo 1. Thus, the observations  $N_i$  under the Poisson model are given by

$$N_i = \sum_j \delta_{x_{ij} + \varepsilon_{ij} - \lfloor x_{ij} + \varepsilon_{ij} \rfloor}$$

where  $\tilde{N}_i = \sum_j \delta_{x_{ij}}$  is the realization of a Poisson point process with intensity function  $\lambda$  and the errors  $\varepsilon_{ij}$  are i.i.d.  $\sim f$ . Note that under the Poisson model each  $N_i$  is again a realization of a Poisson point process whose intensity function is given by the circular convolution  $\lambda \star f$  modulo 1 of  $\lambda$  with the error density  $f$ . More precisely,  $\ell = \lambda \star f$  is given by the formula

$$\ell(t) = \int_0^1 \lambda((t - \varepsilon) - \lfloor t - \varepsilon \rfloor) f(\varepsilon) d\varepsilon, \quad t \in [0, 1). \quad (2.2)$$

By the convolution theorem, we have  $[\ell]_j = [\lambda]_j \cdot [f]_j$  for all  $j \in \mathbb{Z}$ . From Campbell's theorem

(cf. [Ser09], Chapter 3, Theorem 24) it can be deduced that for measurable functions  $g$  we have

$$\mathbb{E} \left[ \int_0^1 g(t) dN_i(t) \right] = \int_0^1 \ell(t) g(t) dt$$

provided that the integral on the right-hand side exists. Exploiting this equation for  $g(t) = \mathbf{e}_j(-t)$  and setting

$$\widehat{[\ell]}_j := \frac{1}{n} \sum_{i=1}^n \int_0^1 \mathbf{e}_j(-t) dN_i(t) \quad (2.3)$$

we thus obtain that  $\mathbb{E}[\widehat{[\ell]}_j] = [\lambda]_j \cdot [f]_j$  for all  $j \in \mathbb{Z}$ . More precisely, we have

$$\widehat{[\ell]}_j = [\lambda]_j \cdot [f]_j + \xi_j \quad \text{for all } j \in \mathbb{Z} \quad (2.4)$$

with centred random variables

$$\xi_j = \widehat{[\ell]}_j - \mathbb{E}[\widehat{[\ell]}_j] = \frac{1}{n} \sum_{i=1}^n \left[ \int_0^1 \mathbf{e}_j(-t) dN_i(t) - \int_0^1 \ell(t) \mathbf{e}_j(-t) dt \right].$$

#### 2.4. Sequence space representation for the Cox model

The Cox model permits a sequence space representation similar to the one of the Poisson model which was obtained in [Big+13]. Under the Cox model, the independent point processes  $N_1, \dots, N_n$  are generated by a random contamination of the individual points of the unobservable Poisson point processes  $\tilde{N}_1, \dots, \tilde{N}_n$  as for the Poisson model. In contrast to the Poisson model, however, the additive error is the same for all the points of one realization  $\tilde{N}_i$ , that is the observations  $N_i$  are given by

$$N_i = \sum_j \delta_{x_{ij} + \varepsilon_i - \lfloor x_{ij} + \varepsilon_i \rfloor} \quad (2.5)$$

where  $\tilde{N}_i = \sum_j \delta_{x_{ij}}$  is the realization of a Poisson point process with intensity function  $\lambda$  and  $\varepsilon_i \sim f$ . Alternatively, the generation of the point processes  $N_i$  can be described by the following two-step procedure: in the first step, random shifts  $\varepsilon_i \sim f$  are generated. In the second step, conditionally on  $\varepsilon_i$ ,  $N_i$  is drawn as the realization of a Poisson point process on  $[0, 1)$  whose intensity function is  $\lambda(t - \varepsilon_i - \lfloor t - \varepsilon_i \rfloor)$ . Thus, in this second model the observations follow the distribution of a Cox process which is directed by the random measure with random intensity  $\lambda(t - \varepsilon - \lfloor t - \varepsilon \rfloor)$  for  $\varepsilon \sim f$ . Note that for  $i = 1, \dots, n$  and integrable functions  $g$  we have

$$\mathbb{E} \left[ \int_0^1 g(t) dN_i(t) \mid \varepsilon_i \right] = \int_0^1 g(t) \lambda(t - \varepsilon_i - \lfloor t - \varepsilon_i \rfloor) dt,$$

which implies

$$\mathbb{E} \left[ \int_0^1 g(t) dN_i(t) \right] = \int_0^1 g(t) \int_0^1 \lambda(t - \varepsilon - \lfloor t - \varepsilon \rfloor) f(\varepsilon) d\varepsilon dt = \int_0^1 g(t) \ell(t) dt,$$

where  $\ell = \lambda \star f$  denotes the circular convolution of the function  $\lambda$  and the density  $f$  as in (2.2). Thus, the mean measure of a generic realization  $N$  obeying the Cox model has the Radon-Nikodym derivative  $\ell$  with respect to the Lebesgue measure. Note that the mean measures of the observed point processes under the Poisson and the Cox model coincide, but the observations for the Cox model stem from a Cox instead of a Poisson process. With  $\widehat{[\ell]}_j$  defined as in (2.3)

the relation

$$\mathbb{E}[\widehat{\ell}]_j \mid \varepsilon_1, \dots, \varepsilon_n = [\lambda]_j \cdot \widetilde{[f]}_j$$

holds with  $\widetilde{[f]}_j = \frac{1}{n} \sum_{i=1}^n \mathbf{e}_j(-\varepsilon_i)$ . Thus, we get the following representation as a sequence space model:

$$\widehat{\ell}]_j = [\lambda]_j \cdot \widetilde{[f]}_j + \xi_j, \quad (2.6)$$

where  $\xi_j = \frac{1}{n} \sum_{i=1}^n [\int_0^1 \mathbf{e}_j(-t) dN_i(t) - \int_0^1 \mathbf{e}_j(-t) \lambda(t - \varepsilon_i - \lfloor t - \varepsilon_i \rfloor) dt]$  are centred random variables for all  $j \in \mathbb{Z}$ . The connection between the sequence space model at hand and the stated sequence space model formulation for statistical linear inverse problems is discussed in detail in Section 2.1 of [Big+13].

### 2.5. Orthonormal series estimator

Recall that each  $\lambda \in \mathbb{L}^2$  can be represented by its Fourier series representation as  $\lambda = \sum_{j \in \mathbb{Z}} [\lambda]_j \mathbf{e}_j$ . Hence, in view of (2.4) and (2.6), a natural estimator of  $\lambda$  is given by

$$\widehat{\lambda}_k = \sum_{0 \leq |j| \leq k} \frac{\widehat{\ell}]_j}{\widetilde{[f]}_j} \mathbf{1}_{\Omega_j} \mathbf{e}_j \quad (2.7)$$

with  $\widehat{\ell}]_j$  as defined in (2.3),  $\widetilde{[f]}_j := \frac{1}{m} \sum_{i=1}^m \mathbf{e}_j(-Y_i)$  and  $\Omega_j := \{|\widetilde{[f]}_j|^2 \geq 1/m\}$ . Note that  $[f]_j$  in (2.4) is not directly available and thus has to be estimated from the sample  $Y_1, \dots, Y_m$  in (1.3). For the Cox model, note that  $\widetilde{[f]}_j$  in (2.6) cannot be observed. In contrast to the setup in [Big+13] we do not assume the density  $f$  to be known and hence  $\widetilde{[f]}_j$  cannot be substituted by its expectation  $[f]_j$  which was suggested in [Big+13]. Since we have the additional sample  $Y_1, \dots, Y_m$  from  $f$  in (1.3) at hand, the most natural idea seems to substitute  $\widetilde{[f]}_j$  by  $\widehat{[f]}_j$  which leads to the same estimator as for the Poisson model. The additional threshold occurring in the definition of  $\widehat{\lambda}_k$  through the indicator function over the set  $\Omega_j$  compensates for 'too small' absolute values of  $\widehat{[f]}_j$  and is imposed in order to avoid unstable behaviour of the estimator. The definition of the event  $\Omega_j$  is in accordance with [NH97] and the chosen threshold corresponds to the parametric rate at which  $[f]_j$  can be estimated from the sample  $Y_1, \dots, Y_m$ . The optimal choice  $k_n^*$  of the dimension parameter in the minimax framework will be determined in Section 3 and depends on the classes  $\Lambda$  and  $\mathcal{F}$ . Thus, the estimator  $\widehat{\lambda}_{k_n^*}$  is not completely data-driven which is unacceptable in applications where the degree of smoothness of the functions  $\lambda$  and  $f$  will usually not be available in advance. The data-driven choice of the dimension parameter is discussed in Sections 4 and 5.

## 3. Minimax theory

### 3.1. Model assumptions

Let  $\gamma = (\gamma_j)_{j \in \mathbb{Z}}$  and  $\alpha = (\alpha_j)_{j \in \mathbb{Z}}$  be strictly positive symmetric sequences and fix  $r > 0, d \geq 1$ . In this section, we derive minimax rates of convergence concerning the maximum risk defined in (2.1) with respect to the classes

$$\Lambda_\gamma^r := \{\lambda \in \mathbb{L}^2 : \lambda \geq 0 \text{ and } \sum_{j \in \mathbb{Z}} \gamma_j |[\lambda]_j|^2 =: \|\lambda\|_\gamma^2 \leq r\}$$

and

$$\mathcal{F}_\alpha^d := \{f \in \mathbb{L}^2 : f \geq 0, [f]_0 = 1 \text{ and } d^{-1} \leq |[f]_j|^2 / \alpha_j \leq d\}$$



of intensity functions and error densities, respectively. The regularity conditions imposed on the sequences  $\gamma$  and  $\alpha$  are summarized in the following assumption.

ASSUMPTION A  $\gamma = (\gamma_j)_{j \in \mathbb{Z}}$ ,  $\alpha = (\alpha_j)_{j \in \mathbb{Z}}$  and  $\omega = (\omega_j)_{j \in \mathbb{Z}}$  are strictly positive symmetric sequences such that  $\gamma_0 = \omega_0 = \alpha_0 = 1$ ,  $\gamma_j \geq 1$  for all  $j \in \mathbb{Z}$  and the sequences  $(\omega_n/\gamma_n)_{n \in \mathbb{N}_0}$  and  $(\alpha_n)_{n \in \mathbb{N}_0}$  are both non-increasing. Finally,  $\rho := \sum_{j \in \mathbb{Z}} \alpha_j < \infty$ .

Note that in the special case  $\omega \equiv 1$ , that is, the weighted norm  $\|\cdot\|_\omega$  coincides with the usual  $\mathbb{L}^2$ -norm, the additional assumption  $\gamma_j \geq 1$  is contained in the requirement that the sequence  $(\omega_n/\gamma_n)_{n \in \mathbb{N}_0}$  is non-increasing.

### 3.2. Minimax lower bounds for the Poisson model

We first derive lower bounds in terms of the sample sizes  $n$  and  $m$  in (1.3) for Poisson model. Our first theorem provides such a lower bound  $\Psi_n$  in terms of the sample size  $n$  under mild assumptions.

THEOREM 3.1 For  $n \in \mathbb{N}$ , set

$$k_n^* := \operatorname{argmin}_{k \in \mathbb{N}_0} \max \left\{ \frac{\omega_k}{\gamma_k}, \sum_{0 \leq |j| \leq k} \frac{\omega_j}{n\alpha_j} \right\} \quad \text{and} \quad \Psi_n := \max \left\{ \frac{\omega_{k_n^*}}{\gamma_{k_n^*}}, \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \right\}. \quad (3.1)$$

Let Assumption A hold, and further assume that

$$(C1) \quad \Gamma := \sum_{j \in \mathbb{Z}} \gamma_j^{-1} < \infty, \text{ and}$$

$$(C2) \quad 0 < \eta^{-1} = \inf_{n \in \mathbb{N}} \Psi_n^{-1} \cdot \min \left\{ \frac{\omega_{k_n^*}}{\gamma_{k_n^*}}, \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \right\} \text{ for some } 1 \leq \eta < \infty.$$

Then, for any  $n \in \mathbb{N}$ ,

$$\inf_{\tilde{\lambda}} \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\tilde{\lambda} - \lambda\|_\omega^2] \geq \frac{\zeta r}{16\eta} \cdot \Psi_n$$

where  $\zeta = \min\{\frac{1}{2\Gamma d\eta}, \frac{2\delta}{d\sqrt{r}}\}$  with  $\delta = \frac{1}{2} - \frac{1}{2\sqrt{2}}$ , and the infimum is taken over all estimators  $\tilde{\lambda}$  of  $\lambda$  based on the observations from (1.3) under the Poisson model.

Before proceeding with the lower bound in  $m$ , let us state some remarks concerning Theorem 3.1. Firstly, it follows immediately from the proof that the lower bound  $\Psi_n$  holds already in case of a known error density. Secondly, assuming the convergence of the series  $\sum_{j \in \mathbb{Z}} \gamma_j^{-1}$  through condition (C1) is necessary only in order to establish the non-negativity of the hypotheses  $\lambda_\theta$  considered in the proof. Finally, it is noteworthy that the proof of Theorem 3.1 can be adapted to the case of direct observations of Poisson point processes with intensity function  $\lambda$  (cf. [Kro16], Theorem 3.3 where the corresponding lower bound is obtained by replacing all the  $\alpha_j$  with 1).

REMARK 3.2 The key ingredient for the proof of Theorem 3.1 is the fact that the Hellinger distance between two PPPs is bounded by the Hellinger distance of the corresponding intensity measures. Since this relation holds only for the special case of Poisson processes, it cannot be directly extended to the Cox model and the given proof fails. Thus, the derivation of minimax lower bounds based on Fourier expansions for the Cox model in our framework remains open and needs to be addressed in future research. Note that [Big+13] give a lower bound proof in case of a known and ordinary smooth error density  $f$ .

The following theorem provides a lower bound  $\Phi_m$  in terms of the sample size  $m$ .



THEOREM 3.3 For  $m \in \mathbb{N}$ , set

$$\Phi_m := \max_{j \in \mathbb{N}} \left\{ \omega_j \gamma_j^{-1} \min \left( 1, \frac{1}{m \alpha_j} \right) \right\}. \quad (3.2)$$

Let Assumption A hold, and in addition assume that

(C3) there exists a density  $f$  in  $\mathcal{F}_\alpha^{\sqrt{d}}$  with  $f \geq 1/2$ .

Then, for any  $m \in \mathbb{N}$ ,

$$\inf_{\tilde{\lambda}} \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\tilde{\lambda} - \lambda\|_\omega^2] \geq \frac{1 - \sqrt{3}/2}{8} \cdot \zeta^2 r d^{-1/2} \cdot \Phi_m$$

where  $\zeta = \min\{\frac{1}{4\sqrt{d}}, 1 - d^{-1/4}\}$  and the infimum is taken over all estimators  $\tilde{\lambda}$  of  $\lambda$  based on the observations from (1.3) under the Poisson model.

Let us state some remarks concerning Theorem 3.3. First, for the proof of the theorem it was sufficient to construct two hypotheses which are statistically indistinguishable but already establish the lower bound  $\Phi_m$ . This is in contrast to the proof of Theorem 3.1 where we had to construct  $2^{k_n^*+1}$  hypotheses. Second, the condition (C3) has to be imposed in order to guarantee that the hypotheses  $f_\theta$  considered in the proof belong to  $\mathcal{F}_\alpha^d$ . It is easy to check that condition (C3) is satisfied if  $\rho_0 := \sum_{j \neq 0} \alpha_j$  satisfies  $\sqrt{d} \geq \max(4\rho_0^2, 1)$ .

REMARK 3.4 The proof of Theorem 3.3 cannot be transferred to the Cox model neither. In the proof, the equality  $\lambda_{-1} \star f_{-1} = \lambda_1 \star f_1$  would imply only the equality of the mean measures of the two Cox process hypotheses but not equality of their distributions (which holds true for PPPs).

The next corollary is an immediate consequence of Theorems 3.1 and 3.3.

COROLLARY 3.5 Under the assumptions of Theorems 3.1 and 3.3, for any  $n, m \in \mathbb{N}$ ,

$$\inf_{\tilde{\lambda}} \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\tilde{\lambda} - \lambda\|_\omega^2] \gtrsim \Psi_n + \Phi_m.$$

### 3.3. Upper bound

Let us now establish an upper bound for the maximum risk in terms of  $n$  and  $m$  for the estimator  $\hat{\lambda}_k$  in (2.7) under a suitable choice of the dimension parameter  $k$ . More precisely, the following theorem establishes an upper bound for the rate of convergence of  $\hat{\lambda}_{k_n^*}$  with  $k_n^*$  defined in (3.1) for the Poisson and the Cox model (an analysis of the proofs indicates only a slightly different numerical constant). Thus, due to the lower bound proofs in the preceding subsection it is shown that  $\hat{\lambda}_{k_n^*}$  attains the minimax rates of convergence in terms of the samples sizes  $n$  and  $m$  under the Poisson model. Note that the optimal choice of the dimension parameter does not depend on the sample size  $m$ .

THEOREM 3.6 Let Assumption A hold and further assume that the samples  $N_1, \dots, N_n$  and  $Y_1, \dots, Y_m$  in (1.3) are drawn in accordance with the Poisson or the Cox model. Then, for any  $n, m \in \mathbb{N}$ ,

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\hat{\lambda}_{k_n^*} - \lambda\|_\omega^2] \lesssim \Psi_n + \Phi_m.$$

$\gamma$	$\alpha$	$\Theta(\Psi_n)$	$\Theta(\Phi_m)$	Restrictions
(pol)	(pol)	$n^{-\frac{2(p-s)}{2p+2a+1}}$	$m^{-\frac{(p-s)\wedge a}{a}}$	$p \geq s, a > \frac{1}{2}$
(exp)	(pol)	$(\log n)^{2s+2a+1} \cdot n^{-1}$	$m^{-1}$	$a > \frac{1}{2}$
(pol)	(exp)	$(\log n)^{-2(p-s)}$	$(\log m)^{-2(p-s)}$	$p \geq s$
(exp)	(exp)	$(\log n)^{2s} \cdot n^{-\frac{p}{p+a}}$	$(\log m)^{2s} \cdot m^{-p/a}$ if $a \geq p$ $m^{-1}$ if $a < p$	

Table 1: Exemplary rates of convergence for nonparametric intensity estimation. The rates are given in the framework of Theorems 3.1, 3.3 and 3.6 which impose the given restrictions. In all examples  $\omega_0 = 1$ ,  $\omega_j = |j|^{2s}$  for  $j \neq 0$ , whereas the choices (pol) and (exp) for the sequences  $\gamma$  and  $\alpha$  are explained in Section 3.4.

### 3.4. Examples of convergence rates

In order to flesh out the abstract results of this section, we consider special choices for the sequences  $\omega$ ,  $\gamma$  and  $\alpha$  and state the resulting rates of convergence with respect to both sample sizes  $n$  and  $m$ . For the sequence  $\omega$ , we will assume throughout that  $\omega_0 = 1$  and  $\omega_j = |j|^{2s}$  for  $j \neq 0$ . The resulting weighted norm corresponds to the usual  $\mathbb{L}^2$ -norm of the  $s^{\text{th}}$  weak derivative.

*Choices for the sequence  $\gamma$ :* Concerning the sequence  $\gamma$  we distinguish the following two scenarios:

(pol):  $\gamma_0 = 0$  and  $\gamma_j = |j|^{2p}$  for all  $j \neq 0$  and some  $p \geq 0$ . This corresponds to the case when the unknown intensity function belongs to some *Sobolev space*.

(exp):  $\gamma_j = \exp(2p|j|)$  for all  $j \in \mathbb{Z}$  and some  $p \geq 0$ . In this case,  $\lambda$  belongs to some space of *analytic functions*.

*Choices for the sequence  $\alpha$ :* Concerning the sequence  $\alpha$  we consider the following scenarios:

(pol):  $\alpha_0 = 0$  and  $\alpha_j = |j|^{-2a}$  for all  $j \neq 0$  and some  $a \geq 0$ . This corresponds to the case when the error density is *ordinary smooth*.

(exp):  $\alpha_j = \exp(-2a|j|)$  for all  $j \in \mathbb{Z}$  and some  $a \geq 0$ .

Table 1 summarises the rates  $\Psi_n$  and  $\Phi_m$  corresponding to the different choices of  $\gamma$  and  $\alpha$ . The rates with respect to  $n$  coincide with the classical rates for nonparametric inverse problems (see for instance Table 1 in [Cav08] where the error variance  $\varepsilon^2$  corresponds to  $n^{-1}$  in our setup and only the case  $s = 0$  is considered).

## 4. Adaptive estimation for the Poisson model

The estimator considered in Theorem 3.6 is obtained by specializing the orthonormal series estimator in (2.7) with  $k_n^*$  defined in (3.1). Thus, this procedure suffers from the apparent drawback that it depends on the smoothness characteristics of both  $\lambda$  and  $f$ , namely on the sequences  $\gamma$  and  $\alpha$ . Since such characteristics are typically unavailable in advance, there is a need for an adaptive selection of the dimension parameter which does not require any *a priori* knowledge on  $\lambda$  and  $f$ . In order to reach such an adaptive definition for the Poisson model (the Cox model will be dealt with in Section 5) we proceed in two steps. In the first step (treated in Subsection 4.1), we assume that the class  $\Lambda_\gamma^r$  is unknown but assume the class  $\mathcal{F}_\alpha^d$  of potential

error densities  $f$  to be known. This assumption allows us to define a *partially adaptive* choice  $\tilde{k}$  of  $k$ . In the second step (treated in Subsection 4.2), we dispense with any knowledge on the smoothness both of  $\lambda$  and  $f$  and propose a *fully adaptive* choice  $\hat{k}$  of the dimension parameter.

#### 4.1. Partially adaptive estimation ( $\Lambda_\gamma^r$ unknown, $\mathcal{F}_\alpha^d$ known)

In this subsection, we aim at choosing  $k$  equal to some  $\tilde{k}$  that, in contrast to  $k_n^*$  in Section 3, does no longer depend on the sequence  $\gamma$  but only on the sequence  $\alpha$ . For the definition of  $\tilde{k}$  some terminology has to be introduced: for any  $k \in \mathbb{N}_0$ , let

$$\Delta_k^\alpha := \max_{0 \leq j \leq k} \omega_j \alpha_j^{-1} \quad \text{and} \quad \delta_k^\alpha := (2k+1) \Delta_k^\alpha \frac{\log(\Delta_k^\alpha \vee (k+3))}{\log(k+3)}.$$

For all  $n, m \in \mathbb{N}$ , setting  $\omega_j^+ := \max_{0 \leq i \leq j} \omega_i$ , we define

$$N_n^\alpha := \inf \left\{ 1 \leq j \leq n : \frac{\alpha_j}{2j+1} < \frac{\log(n+3)\omega_j^+}{n} \right\} - 1 \wedge n,$$

$$M_m^\alpha := \inf \{ 1 \leq j \leq m : \alpha_j < 640dm^{-1} \log(m+1) \} - 1 \wedge m,$$

and set  $K_{nm}^\alpha := N_n^\alpha \wedge M_m^\alpha$ . Now, consider the contrast

$$\Upsilon(t) := \|t\|_\omega^2 - 2\Re \langle \widehat{\lambda}_{n \wedge m}, t \rangle_\omega, \quad t \in \mathbb{L}^2.$$

and define the random sequence of penalties  $(\widetilde{\text{PEN}}_k)_{k \in \mathbb{N}_0}$  via

$$\widetilde{\text{PEN}}_k := \frac{165}{4} d \eta^{-1} \cdot (\widehat{\ell}_0 \vee 1) \cdot \frac{\delta_k^\alpha}{n},$$

where  $\eta \in (0, 1)$  is some additional tuning parameter. The parameter  $\eta$  finds its way into the upper risk bound only as a numerical constant and does not have any effect on the rate of convergence. The dependence of the adaptive estimator on the specific choice of  $\eta$  will be suppressed for the sake of convenience in the sequel. Building on our definition of contrast and penalty, we define the partially adaptive selection of the dimension parameter  $k$  as

$$\tilde{k} := \underset{0 \leq k \leq K_{nm}^\alpha}{\operatorname{argmin}} \{ \Upsilon(\widehat{\lambda}_k) + \widetilde{\text{PEN}}_k \}.$$

The following theorem provides an upper bound for the partially adaptive estimator  $\widehat{\lambda}_{\tilde{k}}$ .

**THEOREM 4.1** *Let Assumption A hold. Then, for any  $n, m \in \mathbb{N}$ ,*

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\widehat{\lambda}_{\tilde{k}} - \lambda\|_\omega^2] \lesssim \min_{0 \leq k \leq K_{nm}^\alpha} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\delta_k^\alpha}{n} \right\} + \Phi_m + \frac{1}{m} + \frac{1}{n}$$

where the observations in (1.3) stem from the Poisson model.

#### 4.2. Fully adaptive estimation ( $\Lambda_\gamma^r$ and $\mathcal{F}_\alpha^d$ unknown)

We now also dispense with the knowledge of the smoothness of the error density  $f$  and propose a fully data-driven selection  $\hat{k}$  of the dimension parameter such that the resulting estimator  $\widehat{\lambda}_{\hat{k}}$  adapts to the unknown smoothness of both  $\lambda$  and  $f$  and attains the optimal rate of convergence in a wide range of scenarios. As in the case of partially adaptive estimation, we have to introduce

some notation first. For  $k \in \mathbb{N}_0$ , let

$$\widehat{\Delta}_k := \max_{0 \leq j \leq k} \frac{\omega_j}{|\widehat{[f]}_j|^2} \mathbf{1}_{\Omega_j} \quad \text{and} \quad \widehat{\delta}_k := (2k+1) \widehat{\Delta}_k \frac{\log(\widehat{\Delta}_k \vee (k+4))}{\log(k+4)}.$$

For  $n, m \in \mathbb{N}$ , set

$$\begin{aligned} \widehat{N}_n &:= \inf\{1 \leq j \leq n : |\widehat{[f]}_j|^2 / (2j+1) < \log(n+4) \omega_j^+ / n\} - 1 \wedge n, \\ \widehat{M}_m &:= \inf\{1 \leq j \leq m : |\widehat{[f]}_j|^2 < m^{-1} \log(m)\} - 1 \wedge m, \end{aligned}$$

and  $\widehat{K}_{nm} := \widehat{N}_n \wedge \widehat{M}_m$ . We consider the same contrast function as in the partially adaptive case but define the random sequence  $(\widehat{\text{PEN}}_k)_{k \in \mathbb{N}_0}$  of penalties now by

$$\widehat{\text{PEN}}_k := 1375 \eta^{-1} \cdot ([\widehat{\ell}]_0 \vee 1) \cdot \frac{\widehat{\delta}_k}{n}.$$

Note that this definition does not depend on the knowledge of the sequence  $\alpha$ . Using this definition of a completely data-driven penalty, we define the fully adaptive selection  $\widehat{k}$  of the dimension parameter  $k$  by means of

$$\widehat{k} := \underset{0 \leq k \leq \widehat{K}_{nm}}{\operatorname{argmin}} \{ \Upsilon(\widehat{\lambda}_k) + \widehat{\text{PEN}}_k \}.$$

In order to state and prove the upper risk bound of the estimator  $\widehat{\lambda}_{\widehat{k}}$ , we have to introduce some further notation. We keep the definition of  $\Delta_k^\alpha$  from Subsection 4.1 but slightly redefine  $\delta_k^\alpha$  as

$$\delta_k^\alpha := (2k+1) \Delta_k^\alpha \frac{\log(\Delta_k^\alpha \vee (k+4))}{\log(k+4)}.$$

For  $k \in \mathbb{N}_0$ , we also define

$$\Delta_k := \max_{0 \leq j \leq k} \frac{\omega_j}{|[f]_j|^2} \quad \text{and} \quad \delta_k := (2k+1) \Delta_k \frac{\log(\Delta_k \vee (k+4))}{\log(k+4)},$$

which can be regarded as analogues of  $\Delta_k^\alpha$  and  $\delta_k^\alpha$  in Subsection 4.1 in the case of a known error density  $f$ . Finally, for  $n, m \in \mathbb{N}$ , define

$$\begin{aligned} N_n^{\alpha-} &:= \inf\{1 \leq j \leq n : \alpha_j / (2j+1) < 4d \log(n+4) \omega_j^+ / n\} - 1 \wedge n, \\ N_n^{\alpha+} &:= \inf\{1 \leq j \leq n : \alpha_j / (2j+1) < \log(n+4) \omega_j^+ / (4dn)\} - 1 \wedge n, \\ M_m^{\alpha-} &:= \inf\{1 \leq j \leq m : \alpha_j < 4dm^{-1} \log m\} - 1 \wedge m, \\ M_m^{\alpha+} &:= \inf\{1 \leq j \leq m : 4d\alpha_j < m^{-1} \log m\} - 1 \wedge m, \end{aligned}$$

and set  $K_{nm}^{\alpha-} := N_n^{\alpha-} \wedge M_m^{\alpha-}$ ,  $K_{nm}^{\alpha+} := N_n^{\alpha+} \wedge M_m^{\alpha+}$ . In contrast to the proof of Theorem 4.1 we have to impose an additional assumption for the proof of an upper risk bound of  $\widehat{\lambda}_{\widehat{k}}$ :

**ASSUMPTION B**  $\exp(-m\alpha_{M_m^{\alpha+}+1}/(128d)) \leq C(\alpha, d)m^{-5}$  for all  $m \in \mathbb{N}$ .

Under this additional assumption, the following theorem establishes a uniform upper risk bound for the completely data-driven estimator.

**THEOREM 4.2** *Let Assumptions A and B hold. Then, for any  $n, m \in \mathbb{N}$ ,*

$$\sup_{\lambda \in \Lambda_\gamma} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\widehat{\lambda}_{\widehat{k}} - \lambda\|_\omega^2] \lesssim \min_{0 \leq k \leq K_{nm}^{\alpha-}} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\delta_k^\alpha}{n} \right\} + \Phi_m + \frac{1}{m} + \frac{1}{n}$$

where the observations in (1.3) stem from the Poisson model.

Note that the only additional prerequisite of Theorem 4.2 in contrast to 4.1 is the validity of Assumption B.

#### 4.3. Examples of convergence rates (continued from Subsection 3.4)

We consider the same configurations for the sequences  $\omega$ ,  $\gamma$  and  $\alpha$  as in Subsection 3.4. In particular, we assume that  $\omega_0 = 1$  and  $\omega_j = |j|^{2s}$  for all  $j \neq 0$ . The different configurations for  $\gamma$  and  $\alpha$  will be investigated in the following (compare also with the minimax rates of convergence given in Table 1). Note that the additional Assumption B is satisfied in all the considered cases. Let us define  $k_n^\diamond := \operatorname{argmin}_{k \in \mathbb{N}_0} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\delta_k^\alpha}{n} \right\}$ , that is,  $k_n^\diamond$  realizes the best compromise between squared bias and penalty.

*Scenario (pol)-(pol):* In this scenario, it holds  $k_n^\diamond \asymp n^{\frac{1}{2p+2a+1}}$  and  $N_n^{\alpha-} \asymp (n/\log n)^{\frac{1}{2s+2a+1}}$ . First assume that  $N_n^{\alpha-} \leq M_m^{\alpha-}$ . In case that  $s < p$ , the rate w.r.t.  $n$  is  $n^{-\frac{2(p-s)}{2p+2a+1}}$  which is the minimax optimal rate. In case that  $s = p$ , it holds  $N_n^{\alpha-} \lesssim k_n^\diamond$  and the rate is  $(n/\log n)^{-\frac{2(p-s)}{2p+2a+1}}$  which is minimax optimal up to a logarithmic factor. Assume now that  $M_m^{\alpha-} \leq N_n^{\alpha-}$ . If  $k_n^\diamond \lesssim M_m^{\alpha-}$ , then the estimator obtains the optimal rate with respect to  $n$ . Otherwise,  $M_m^{\alpha-} \asymp (m/\log m)^{1/(2a)}$  yields the contribution  $(m/\log m)^{-\frac{p-s}{a}}$  to the rate.

*Scenario (exp)-(pol):*  $N_n^{\alpha-} \asymp (n/\log n)^{1/(2a+2s+1)}$  as in scenario (pol)-(pol). Since  $k_n^\diamond \asymp \log n$ , it holds  $k_n^\diamond \lesssim N_n^{\alpha-}$  and the optimal rate with respect to  $n$  holds in case that  $k_n^\diamond \lesssim M_m^{\alpha-}$ . Otherwise, the bias-penalty tradeoff generates the contribution  $(M_m^{\alpha-})^{2s} \cdot \exp(-2p \cdot M_m^{\alpha-})$  to the rate.

*Scenario (pol)-(exp):* It holds that  $k_n^\diamond \asymp N_n^{\alpha-}$  and again the sample size  $n$  is no obstacle for attaining the optimal rate of convergence. If  $k_n^\diamond \lesssim M_m^{\alpha-}$ , the optimal rate holds as well. If  $M_m^{\alpha-} \lesssim k_n^\diamond$ , we get the rate  $(\log m)^{-2(p-s)}$  which coincides with the optimal rate with respect to the sample size  $m$ .

*Scenario (exp)-(exp):* We have  $N_n^{\alpha-} \asymp \log n$  and  $k_1 \leq k_n^\diamond \leq k_2$  where  $k_1$  is the solution of  $k_1^2 \exp((2a+2p)k_1) \asymp n$  and  $k_2$  the solution of  $\exp((2a+2p)k_2) \asymp n$ . Thus, we have  $k_n^\diamond \lesssim N_n^{\alpha-}$  and computation of  $\frac{\omega_{k_1}}{\gamma_{k_1}}$  resp.  $\delta_{k_2}^\alpha/n$  shows that only a loss by a logarithmic factor can occur as far as  $k_n^\diamond \leq N_n^{\alpha-} \wedge M_m^{\alpha-}$ . If  $M_m^{\alpha-} \leq k_n^\diamond$ , the contribution to the rate arising from the trade-off between squared bias and penalty is determined by  $(M_m^{\alpha-})^{2s} \cdot \exp(-2pM_m^{\alpha-})$  which deteriorates the optimal rate with respect to  $m$  at most by a logarithmic factor.

**REMARK 4.3** In this paper, we have not considered the case that the Fourier coefficients of the error density obey a power-exponential decay, that is  $\alpha_j = \exp(-2\kappa|j|^a)$  for some  $\kappa > 0$  and arbitrary  $a > 0$ . Indeed, for our definition of the quantity  $M_m^{\alpha+}$ , Assumption B is not satisfied in this case. This shortage can be removed by considering a more elaborate choice of the quantities  $\widehat{M}_m$  and  $M_m^{\alpha+}$  as was considered in [JS13] but we do not include this here.

## 5. Adaptive estimation for the Cox model

Unfortunately, the approach from Section 4 cannot be transferred in order to obtain an upper risk bound for an adaptive estimator in the case of Cox observations. Thus, in this section we follow another approach. The price we have to pay is that we can only obtain rates that contain

an additional logarithmic factor. Again we split our investigation into the partially adaptive and the fully adaptive case.

### 5.1. Partially adaptive case

We define  $\mathbb{D}_k^\alpha := \sum_{0 \leq |j| \leq k} \frac{\omega_j}{\alpha_j}$  which might be interpreted as the dimension of the model corresponding to the linear subspace of  $\mathbb{L}^2$  spanned by the  $\mathbf{e}_j$  with  $j \in \{-k, \dots, k\}$  in the considered inverse problem. In addition, we define the quantities  $N_n^\alpha$ ,  $M_m^\alpha$ , and  $K_{nm}^\alpha$  as well as the contrast function  $\Upsilon$  exactly as in Section 4.1. However, we replace the definition of the penalty given for the case of Poisson observations with

$$\widetilde{\text{PEN}}_k := 2000\eta^{-1} \cdot (\widehat{[\ell]}_0 \vee 1) \cdot \frac{d\mathbb{D}_k^\alpha \log(n+2)}{n} + 2000\eta^{-2} \cdot (\widehat{[\ell]}_0^2 \vee 1) \cdot \frac{d\mathbb{D}_k^\alpha \log(n+2)}{n}$$

where  $\eta \in (0, 1)$ . Based on this updated definition of penalty we define the adaptive choice of the dimension parameter in the case of Cox observations by means of

$$\tilde{k} := \underset{0 \leq j \leq K_{nm}^\alpha}{\operatorname{argmin}} \{ \Upsilon(\widehat{\lambda}_k) + \widetilde{\text{PEN}}_k \}.$$

**THEOREM 5.1** *Let Assumption A hold. Then, for any  $n, m \in \mathbb{N}$ ,*

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2] \lesssim \min_{0 \leq k \leq K_{nm}^\alpha} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\mathbb{D}_k^\alpha \log(n+2)}{n} \right\} + \Phi_m + \frac{1}{m} + \frac{1}{n}$$

where the observations in (1.3) stem from the Cox model.

Note that the proof of Theorem 5.1 is more intricate than the one of Theorem 4.1 due to the fact that we need to introduce some additional terms in the proof. In order to deal with these terms we have to apply Talagrand type concentration inequalities both for Poisson processes and random variables.

### 5.2. Fully adaptive case

In the fully adaptive case, we replace the dimension parameter  $\mathbb{D}_k^\alpha$  from the previous subsection by the estimate

$$\widehat{\mathbb{D}}_k := \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|\widehat{[f]}_j|^2} \mathbf{1}_{\Omega_j}.$$

Moreover, as in the Poisson case, we have to adapt the definition of the penalty

$$\widehat{\text{PEN}}_k := 8000\eta^{-1} \cdot (\widehat{[\ell]}_0 \vee 1) \cdot \frac{\widehat{\mathbb{D}}_k \log(n+2)}{n} + 8000\eta^{-2} \cdot (\widehat{[\ell]}_0^2 \vee 1) \cdot \frac{\widehat{\mathbb{D}}_k \log(n+2)}{n}.$$

We define the contrast function  $\Upsilon$  exactly as in Subsection 4.1. For  $n, m \in \mathbb{N}$ , set

$$\begin{aligned} \widehat{N}_n &:= \inf\{1 \leq j \leq n : |\widehat{[f]}_j|^2 / (2j+1) < \log(n+3)\omega_j^+ / n\} - 1 \wedge n \\ \widehat{M}_m &:= \inf\{1 \leq j \leq m : |\widehat{[f]}_j|^2 < m^{-1} \log(m)\} - 1 \wedge m, \end{aligned}$$

and  $\widehat{K}_{nm} := \widehat{N}_n \wedge \widehat{M}_m$ . We define the fully data-driven choice  $\widehat{k}$  of  $k$  in analogy to the approach for the Poisson model via

$$\widehat{k} := \underset{0 \leq k \leq \widehat{K}_{nm}}{\operatorname{argmin}} \{ \Upsilon(\widehat{\lambda}_k) + \widehat{\text{PEN}}_k \}.$$

For the statement and the proof of the following theorem, define for  $n, m \in \mathbb{N}$  the quantities

$$\begin{aligned} N_n^{\alpha-} &:= \inf\{1 \leq j \leq n : \alpha_j/(2j+1) < 4d \log(n+3) \omega_j^+/n\} - 1 \wedge n, \\ N_n^{\alpha+} &:= \inf\{1 \leq j \leq n : \alpha_j/(2j+1) < \log(n+3) \omega_j^+/(4dn)\} - 1 \wedge n, \\ M_m^{\alpha-} &:= \inf\{1 \leq j \leq m : \alpha_j < 4dm^{-1} \log m\} - 1 \wedge m, \\ M_m^{\alpha+} &:= \inf\{1 \leq j \leq m : 4d\alpha_j < m^{-1} \log m\} - 1 \wedge m, \end{aligned}$$

$K_{nm}^{\alpha-} := N_n^{\alpha-} \wedge M_m^{\alpha-}$ , and  $K_{nm}^{\alpha+} := N_n^{\alpha+} \wedge M_m^{\alpha+}$ . Note that the proof of the following theorem requires the validity of Assumption B again.

**THEOREM 5.2** *Let Assumptions A and B hold. Then, for any  $n, m \in \mathbb{N}$ ,*

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\hat{\lambda}_k - \lambda\|_\omega^2] \lesssim \min_{0 \leq k \leq K_{nm}^{\alpha-}} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\mathbb{D}_k^\alpha \log(n+2)}{n} \right\} + \Phi_m + \frac{1}{m} + \frac{1}{n}$$

where the observations in (1.3) stem from the Cox model.

**REMARK 5.3** Using the approach presented in this subsection we are not able to dispense with the additional logarithmic factor in the rates in case of the Cox model. Note that in case that the error density  $f$  is known (which is informally equivalent to  $m = \infty$ ) we regain the adaptive rate established in [Big+13] for the case that the unknown intensity is ordinary smooth and the Fourier coefficients of  $f$  obey a polynomial decay. However, our results are more general since we do not exclusively consider the case of polynomially decreasing Fourier coefficients.

**REMARK 5.4** Needless to say, the numerical constants in the definition of the penalty are ridiculously large which makes our rate optimal estimator hardly applicable for small sample sizes. Hence there is still research necessary to establish an estimator which performs well both from a theoretical point of view and also yields good results for simulations with relatively small sample sizes. Another approach would be to calibrate numerical constants in the penalty by means of a simulation study as way done for example in [CRT06].

**REMARK 5.5** Of course, the approach presented in this subsection can also be applied to the case of Poisson observations but since the logarithmic factor in the rates is unavoidable we would obtain worse rates than using the approach from Section 4.

### 5.3. Examples of convergence rates (continued from Subsections 3.4 and 4.3)

Note that in all the scenarios considered in Table 1 we have  $k_n^\diamond \lesssim N_n^{\alpha-}$  where  $k_n^\diamond$  denotes the optimal trade-off between the squared bias  $\omega_k/\gamma_k$  and the term  $\mathbb{D}_k^\alpha \log(n+2)/n$ . Computations similar to the ones leading to the rates in Table 1 show that the rates with respect to the sample size  $n$  are those from the minimax framework in Table 1 with  $n$  replaced with  $n/\log(n+2)$  as long as  $k_n^\diamond \leq N_n^{\alpha-} \wedge M_m^{\alpha-}$ . If  $M_m^{\alpha-} \leq k_n^\diamond$ ,  $M_m^{\alpha-}$  determines the rate exactly with the same contribution as in 4.3. It seems noteworthy to mention that in the scenario (pol)-(exp) we attain the upper bound from the minimax theory in Theorem 3.6. In the case (pol)-(pol), however, one can observe a loss by a logarithmic factor which was also observed in case of adaptive estimation in [Big+13].



## A. Proofs of Section 3

### A.1. Proof of Theorem 3.1

Let us define  $\zeta$  as in the statement of the theorem and for each  $\theta = (\theta_j)_{0 \leq j \leq k_n^*} \in \{\pm 1\}^{k_n^*+1}$  the function  $\lambda_\theta$  through

$$\begin{aligned}\lambda_\theta &:= \left(\frac{r}{4}\right)^{1/2} + \theta_0 \left(\frac{r\zeta}{4n}\right)^{1/2} + \left(\frac{r\zeta}{4n}\right)^{1/2} \sum_{1 \leq |j| \leq k_n^*} \theta_{|j|} \alpha_j^{-1/2} \mathbf{e}_j \\ &= \left(\frac{r}{4}\right)^{1/2} + \left(\frac{r\zeta}{4n}\right)^{1/2} \sum_{0 \leq |j| \leq k_n^*} \theta_{|j|} \alpha_j^{-1/2} \mathbf{e}_j.\end{aligned}$$

Then each  $\lambda_\theta$  is a real-valued function by definition which is non-negative since we have

$$\begin{aligned}\left\| \left(\frac{r\zeta}{4n}\right)^{1/2} \sum_{0 \leq |j| \leq k_n^*} \theta_{|j|} \alpha_j^{-1/2} \mathbf{e}_j \right\|_\infty &\leq \left(\frac{r\zeta}{4n}\right)^{1/2} \sum_{0 \leq |j| \leq k_n^*} \alpha_j^{-1/2} \\ &\leq \left(\frac{r\zeta}{4}\right)^{1/2} \left( \sum_{0 \leq |j| \leq k_n^*} \gamma_j^{-1} \right)^{1/2} \left( \sum_{0 \leq |j| \leq k_n^*} \frac{\gamma_j}{n\alpha_j} \right)^{1/2} \\ &\leq \left(\frac{r\zeta\Gamma}{4}\right)^{1/2} \left( \frac{\gamma_{k_n^*}^*}{\omega_{k_n^*}^*} \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \right)^{1/2} \\ &\leq \left(\frac{r\zeta\eta\Gamma}{4}\right)^{1/2} \leq \left(\frac{r}{4}\right)^{1/2}.\end{aligned}$$

Moreover  $\|\lambda_\theta\|_\gamma^2 \leq r$  holds for each  $\theta \in \{\pm 1\}^{k_n^*+1}$  due to the estimate

$$\begin{aligned}\|\lambda_\theta\|_\gamma^2 &= \sum_{0 \leq |j| \leq k_n^*} |[\lambda_\theta]_j|^2 \gamma_j = \left[ \left(\frac{r}{4}\right)^{1/2} + \theta_0 \left(\frac{r\zeta}{4n}\right)^{1/2} \right]^2 + \frac{r\zeta}{4} \sum_{1 \leq |j| \leq k_n^*} \frac{\gamma_j}{n\alpha_j} \\ &\leq \frac{r}{2} + \frac{r\zeta}{2n} + \frac{r\zeta}{4} \frac{\gamma_{k_n^*}^*}{\omega_{k_n^*}^*} \sum_{1 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \\ &\leq \frac{r}{2} + \frac{r\zeta}{2} \frac{\gamma_{k_n^*}^*}{\omega_{k_n^*}^*} \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \leq r.\end{aligned}$$

This estimate and the non-negativity of  $\lambda_\theta$  together imply  $\lambda_\theta \in \Lambda_\gamma^r$  for all  $\theta \in \{\pm 1\}^{k_n^*+1}$ . From now on let  $f \in \mathcal{F}_\alpha^d$  be fixed and let  $\mathbb{P}_\theta$  denote the joint distribution of the i.i.d. samples  $N_1, \dots, N_n$  and  $Y_1, \dots, Y_m$  when the true parameters are  $\lambda_\theta$  and  $f$ , respectively. Let  $\mathbb{P}_\theta^{N_i}$  denote the corresponding one-dimensional marginal distributions and  $\mathbb{E}_\theta$  the expectation with respect to  $\mathbb{P}_\theta$ . Let  $\tilde{\lambda}$  be an arbitrary estimator of  $\lambda$ . The key argument of the proof is the following reduction scheme:

$$\begin{aligned}\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E} \|\tilde{\lambda} - \lambda\|_\omega^2 &\geq \sup_{\theta \in \{\pm 1\}^{k_n^*+1}} \mathbb{E}_\theta [\|\tilde{\lambda} - \lambda_\theta\|_\omega^2] \geq \frac{1}{2^{k_n^*+1}} \sum_{\theta \in \{\pm 1\}^{k_n^*+1}} \mathbb{E}_\theta [\|\tilde{\lambda} - \lambda_\theta\|_\omega^2] \\ &= \frac{1}{2^{k_n^*+1}} \sum_{\theta \in \{\pm 1\}^{k_n^*+1}} \sum_{0 \leq |j| \leq k_n^*} \omega_j \mathbb{E}_\theta [|\tilde{\lambda} - \lambda_\theta|_j|^2]\end{aligned}$$

$$= \frac{1}{2^{k_n^*+1}} \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{2} \sum_{\theta \in \{\pm 1\}^{k_n^*+1}} \{ \mathbb{E}_\theta[|\tilde{\lambda} - \lambda_\theta|_j|^2] + \mathbb{E}_{\theta^{(|j|)}}[|\tilde{\lambda} - \lambda_{\theta^{(|j|)}}|_j|^2] \}, \quad (\text{A.1})$$

where for  $\theta \in \{\pm 1\}^{k_n^*+1}$  and  $j \in \{-k_n^*, \dots, k_n^*\}$  the element  $\theta^{(|j|)} \in \{\pm 1\}^{k_n^*+1}$  is defined by  $\theta_k^{(|j|)} = \theta_k$  for  $k \neq |j|$  and  $\theta_{|j|}^{(|j|)} = -\theta_{|j|}$ . Consider the Hellinger affinity  $\rho(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}}) := \int \sqrt{d\mathbb{P}_\theta d\mathbb{P}_{\theta^{(|j|)}}}$ . For an arbitrary estimator  $\tilde{\lambda}$  of  $\lambda$  we have

$$\begin{aligned} \rho(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}}) &\leq \int \frac{|\tilde{\lambda} - \lambda_\theta|_j|}{|[\lambda_\theta - \lambda_{\theta^{(|j|)}}]_j|} \sqrt{d\mathbb{P}_\theta d\mathbb{P}_{\theta^{(|j|)}}} + \int \frac{|[\tilde{\lambda} - \lambda_{\theta^{(|j|)}}]_j|}{|[\lambda_\theta - \lambda_{\theta^{(|j|)}}]_j|} \sqrt{d\mathbb{P}_\theta d\mathbb{P}_{\theta^{(|j|)}}} \\ &\leq \left( \int \frac{|\tilde{\lambda} - \lambda_\theta|_j|^2}{|[\lambda_\theta - \lambda_{\theta^{(|j|)}}]_j|^2} d\mathbb{P}_\theta \right)^{1/2} + \left( \int \frac{|[\tilde{\lambda} - \lambda_{\theta^{(|j|)}}]_j|^2}{|[\lambda_\theta - \lambda_{\theta^{(|j|)}}]_j|^2} d\mathbb{P}_{\theta^{(|j|)}} \right)^{1/2} \end{aligned}$$

from which we conclude by means of the elementary inequality  $(a+b)^2 \leq 2a^2 + 2b^2$  that

$$\frac{1}{2} |[\lambda_\theta - \lambda_{\theta^{(|j|)}}]_j|^2 \rho^2(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}}) \leq \mathbb{E}_\theta[|\tilde{\lambda} - \lambda_\theta|_j|^2] + \mathbb{E}_{\theta^{(|j|)}}[|\tilde{\lambda} - \lambda_{\theta^{(|j|)}}|_j|^2].$$

Introduce the Hellinger distance between two probability measures  $\mathbb{P}$  and  $\mathbb{Q}$  as  $H(\mathbb{P}, \mathbb{Q}) := (\int [\sqrt{d\mathbb{P}} - \sqrt{d\mathbb{Q}}]^2)^{1/2}$  and, analogously, the Hellinger distance between two finite measures  $\nu$  and  $\mu$  (that not necessarily have total mass equal to one) by  $H(\nu, \mu) := (\int [\sqrt{d\nu} - \sqrt{d\mu}]^2)^{1/2}$  (as usual, the integral is formed with respect to any measure dominating both  $\nu$  and  $\mu$ ). Let  $\nu_\theta$  denote the intensity measure of a Poisson point process  $N$  on  $[0, 1)$  whose Radon-Nikodym derivative with respect to the Lebesgue measure is given by  $\ell_\theta := \lambda_\theta \star f$ . Note that we have the estimate  $\ell_\theta \geq \delta \sqrt{r}$  for all  $\theta \in \{\pm 1\}^{k_n^*+1}$  with  $\delta = \frac{1}{2} - \frac{1}{2\sqrt{2}}$  due to

$$\left( \frac{r\zeta}{4n} \right)^{1/2} + \sum_{1 \leq |j| \leq k_n^*} |[\lambda_\theta]_j \cdot [f]_j| \leq \left( \frac{rd\zeta}{4n} \right)^{1/2} \sum_{0 \leq |j| \leq k_n^*} \alpha_j^{-1/2} \leq \frac{\sqrt{r}}{2\sqrt{2}}$$

which can be realized in analogy to the non-negativity of  $\lambda_\theta$  shown above. We obtain

$$H^2(\nu_\theta, \nu_{\theta^{(|j|)}}) = \int (\sqrt{\ell_\theta} - \sqrt{\ell_{\theta^{(|j|)}}})^2 = \int \frac{|\ell_\theta - \ell_{\theta^{(|j|)}}|^2}{(\sqrt{\ell_\theta} + \sqrt{\ell_{\theta^{(|j|)}}})^2} \leq \frac{\|\ell_\theta - \ell_{\theta^{(|j|)}}\|_2^2}{4\delta\sqrt{r}} = \frac{\zeta d\sqrt{r}}{4\delta n} \leq \frac{1}{n}.$$

Since the distribution of the sample  $Y_1, \dots, Y_m$  does not depend on the choice of  $\theta$  we obtain

$$H^2(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}}) \leq \sum_{i=1}^n H^2(\mathbb{P}_\theta^{N_i}, \mathbb{P}_{\theta^{(|j|)}}^{N_i}) \leq \sum_{i=1}^n H^2(\nu_\theta, \nu_{\theta^{(|j|)}}) \leq 1,$$

where the first estimate follows from Lemma 3.3.10 (i) in [Rei89] and the second one is due to Theorem 3.2.1 in [Rei93] which can be applied since each  $N_i$  is a Poisson point process for the Poisson model. Thus, the relation  $\rho(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}}) = 1 - \frac{1}{2} H^2(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}})$  implies  $\rho(\mathbb{P}_\theta, \mathbb{P}_{\theta^{(|j|)}}) \geq \frac{1}{2}$ . Finally, putting the obtained estimates into the reduction scheme (A.1) leads to

$$\begin{aligned} \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E} \|\tilde{\lambda} - \lambda\|_\omega^2 &\geq \frac{1}{2^{k_n^*+1}} \sum_{\theta \in \{\pm 1\}^{k_n^*+1}} \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{2} \{ \mathbb{E}_\theta[|\tilde{\lambda} - \lambda_\theta|_j|^2] + \mathbb{E}_{\theta^{(|j|)}}[|\tilde{\lambda} - \lambda_{\theta^{(|j|)}}|_j|^2] \} \\ &\geq \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{16} |[\lambda_\theta - \lambda_{\theta^{(|j|)}}]_j|^2 = \frac{\zeta r}{16} \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \geq \frac{\zeta r}{16\eta} \cdot \Psi_n \end{aligned}$$

which finishes the proof of the theorem since  $\tilde{\lambda}$  was arbitrary.  $\square$

### A.2. Proof of Theorem 3.3

The following reduction scheme follows along a general strategy that is well-known for the establishment of lower bounds in nonparametric estimation (for a detailed account cf. [Tsy08], Chapter 2). Note that by Markov's inequality we have for an arbitrary estimator  $\tilde{\lambda}$  of  $\lambda$  and an arbitrary  $A > 0$  (which will be specified below)

$$\mathbb{E}[\Phi_m^{-1} \|\tilde{\lambda} - \lambda\|_\omega^2] \geq A \cdot \mathbb{P}(\|\tilde{\lambda} - \lambda\|_\omega^2 \geq A\Phi_m),$$

which by reduction to two hypotheses implies

$$\begin{aligned} \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\Phi_m^{-1} \|\tilde{\lambda} - \lambda\|_\omega^2] &\geq A \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{P}(\|\tilde{\lambda} - \lambda\|_\omega^2 \geq A\Phi_m) \\ &\geq A \sup_{\theta \in \{\pm 1\}} \mathbb{P}_\theta(\|\tilde{\lambda} - \lambda_\theta\|_\omega^2 \geq A\Phi_m) \end{aligned}$$

where  $\mathbb{P}_\theta$  denotes the distribution when the true parameters are  $\lambda_\theta$  and  $f_\theta$ . The specific hypotheses  $\lambda_1, \lambda_{-1}$  and  $f_1, f_{-1}$  will be specified below. If  $\lambda_{-1}$  and  $\lambda_1$  can be chosen such that  $\|\lambda_1 - \lambda_{-1}\|_\omega^2 \geq 4A\Phi_m$ , application of the triangle inequality yields

$$\mathbb{P}_\theta(\|\tilde{\lambda} - \lambda_\theta\|_\omega^2 \geq A\Phi_m) \geq \mathbb{P}_\theta(\tau^* \neq \theta)$$

where  $\tau^*$  denotes the *minimum distance test* defined through  $\tau^* = \arg \min_{\theta \in \{\pm 1\}} \|\tilde{\lambda} - \lambda_\theta\|_\omega^2$ . Hence, we obtain

$$\begin{aligned} \inf_{\tilde{\lambda}} \sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{P}(\|\tilde{\lambda} - \lambda\|_\omega^2 \geq A\Phi_m) &\geq \inf_{\tilde{\lambda}} \sup_{\theta \in \{\pm 1\}} \mathbb{P}_\theta(\|\tilde{\lambda} - \lambda_\theta\|_\omega^2 \geq A\Phi_m) \\ &\geq \inf_{\tau} \sup_{\theta \in \{\pm 1\}} \mathbb{P}_\theta(\tau \neq \theta) \\ &=: p^*, \end{aligned} \tag{A.2}$$

where the infimum is taken over all  $\{\pm 1\}$ -valued functions  $\tau$  based on the observations. Thus, it remains to find hypotheses  $\lambda_1, \lambda_{-1} \in \Lambda_\gamma^r$  and  $f_1, f_{-1} \in \mathcal{F}_\alpha^d$  such that

$$\|\lambda_1 - \lambda_{-1}\|_\omega^2 \geq 4A\Phi_m, \tag{A.3}$$

and which allow us to bound  $p^*$  by a universal constant (independent of  $m$ ) from below.

For this purpose, set  $k_m^* := \arg \max_{j \geq 1} \{\frac{\omega_j}{\gamma_j} \min(1, \frac{1}{m\alpha_j})\}$  and  $a_m := \zeta \min(1, m^{-1/2} \alpha_{k_m^*}^{-1/2})$ , where  $\zeta$  is defined as in the statement of the theorem. Take note of the inequalities

$$1/d^{1/2} = (1 - (1 - 1/d^{1/4}))^2 \leq (1 - a_m)^2 \leq 1$$

and

$$1 \leq (1 + a_m)^2 \leq (1 + (1 - 1/d^{1/4}))^2 = (2 - 1/d^{1/4})^2 \leq d^{1/2}$$

which in combination imply  $1/d^{1/2} \leq (1 + \theta a_m)^2 \leq d^{1/2}$  for  $\theta \in \{\pm 1\}$ . These inequalities will be used below without further reference. For  $\theta \in \{\pm 1\}$ , we define

$$\lambda_\theta = \left(\frac{r}{2}\right)^{1/2} + (1 - \theta a_m) \left(\frac{r}{8}\right)^{1/2} d^{-1/4} \gamma_{k_m^*}^{-1/2} (\mathbf{e}_{k_m^*} + \mathbf{e}_{-k_m^*}).$$

Note that  $\lambda_\theta$  is real-valued by definition. Furthermore, we have

$$\|\lambda_\theta\|_\gamma^2 = \frac{r}{2} + 2\gamma_{k_m^*} |[\lambda_\theta]_{k_m^*}|^2 \leq \frac{r}{2} + (1 + a_m)^2 \frac{r}{4} d^{-1/2} \leq \frac{3r}{4}$$

and

$$|\lambda_\theta(t)| \geq \left(\frac{r}{2}\right)^{1/2} - 2\left(\frac{r}{8}\right)^{1/2} \geq 0 \quad \forall t \in [0, 1),$$

which together imply that  $\lambda_\theta \in \Lambda_\gamma^r$  for  $\theta \in \{\pm 1\}$ . The identity

$$\|\lambda_1 - \lambda_{-1}\|_\omega^2 = r a_m^2 d^{-1/2} \omega_{k_m^*} \gamma_{k_m^*}^{-1} = \zeta^2 r d^{-1/2} \cdot \Phi_m$$

shows that the condition in (A.3) is satisfied with  $A = \zeta^2 r / (4\sqrt{d})$ .

Let  $f \in \mathcal{F}_\alpha^{\sqrt{d}}$  be such that  $f \geq 1/2$  (the existence is guaranteed through condition (C4)) and define for  $\theta \in \{\pm 1\}$

$$f_\theta = f + \theta a_m ([f]_{k_m^*} \mathbf{e}_{k_m^*} + [f]_{-k_m^*} \mathbf{e}_{-k_m^*}).$$

Since  $k_m^* \geq 1$  we have  $\int_0^1 f_\theta(x) dx = 1$  and  $f_\theta \geq 0$  holds because of the estimate

$$|f_\theta(t)| \geq 1/2 - 2a_m \alpha_{k_m^*}^{1/2} d^{1/2} \geq 0 \quad \text{for all } t.$$

For  $|j| \neq k_m^*$ , we have  $[f]_j = [f_\theta]_j$  and thus trivially  $1/d \leq |[f_\theta]_j|^2 / \alpha_j \leq d$  for  $|j| \neq k_m^*$  since  $\mathcal{F}_\alpha^{\sqrt{d}} \subset \mathcal{F}_\alpha^d$ . Moreover

$$1/d \leq d^{-1/2} \frac{|[f]_{\pm k_m^*}|^2}{\alpha_{\pm k_m^*}} \leq \frac{(1 + \theta a_m)^2 |[f]_{\pm k_m^*}|^2}{\alpha_{\pm k_m^*}} \leq d^{1/2} \frac{|[f]_{\pm k_m^*}|^2}{\alpha_{\pm k_m^*}} \leq d$$

and hence  $f_\theta \in \mathcal{F}_\alpha^d$  for  $\theta \in \{\pm 1\}$ .

To obtain a lower bound for  $p^*$  defined in (A.2) consider the joint distribution  $\mathbb{P}_\theta$  of the samples  $N^1, \dots, N^n$  and  $Y_1, \dots, Y_m$  under  $\lambda_\theta$  and  $f_\theta$ . Note that due to our construction we have  $\lambda_{-1} \star f_{-1} = \lambda_1 \star f_1$ . Thus  $\mathbb{P}_{-1}^{N^i} = \mathbb{P}_1^{N^i}$  for all  $i = 1, \dots, n$  (due to the fact that the distribution of a *Poisson* point process is determined by its intensity) and the Hellinger distance between  $\mathbb{P}_{-1}$  and  $\mathbb{P}_1$  does only depend on the distribution of the sample  $Y_1, \dots, Y_m$ . More precisely,

$$H^2(\mathbb{P}_{-1}, \mathbb{P}_1) = H^2(\mathbb{P}_{-1}^{Y_1, \dots, Y_m}, \mathbb{P}_1^{Y_1, \dots, Y_m}) \leq m H^2(\mathbb{P}_{-1}^{Y_1}, \mathbb{P}_1^{Y_1}),$$

and we proceed by bounding  $H^2(\mathbb{P}_{-1}^{Y_1}, \mathbb{P}_1^{Y_1})$  from above. Recall that  $f \geq 1/2$  which is used to obtain the estimate

$$H^2(\mathbb{P}_{-1}^{Y_1}, \mathbb{P}_1^{Y_1}) = \int_0^1 \frac{|f_1(x) - f_{-1}(x)|^2}{2f(x)} dx \leq \int |f_1(x) - f_{-1}(x)|^2 dx \leq 8 d a_m^2 \alpha_{k_m^*} \leq \frac{1}{m}.$$

Hence we have  $H^2(\mathbb{P}_{-1}, \mathbb{P}_1) \leq 1$  and application of statement (ii) of Theorem 2.2 in [Tsy08] with  $\alpha = 1$  implies  $p^* \geq \frac{1}{2}(1 - \sqrt{3}/2)$  which finishes the proof of the theorem.  $\square$

### A.3. Proof of Theorem 3.6

We give the proof for the Poisson model only. The proof for the Cox model follows in complete analogy by exploiting statement ii) instead of i) in part a) of Lemma A.1.

Set  $\lambda_{k_n^*} := \sum_{0 \leq |j| \leq k_n^*} [\lambda]_j \mathbf{1}_{\Omega_j} \mathbf{e}_j$ . The proof consists in finding appropriate upper bounds for the quantities  $\square$  and  $\triangle$  in the estimate

$$\mathbb{E}[\|\hat{\lambda}_{k_n^*} - \lambda\|_\omega^2] \leq 2 \mathbb{E}[\|\hat{\lambda}_{k_n^*} - \tilde{\lambda}_{k_n^*}\|_\omega^2] + 2 \mathbb{E}[\|\lambda - \tilde{\lambda}_{k_n^*}\|_\omega^2] =: 2\square + 2\triangle. \quad (\text{A.4})$$

Upper bound for  $\square$ : Using the identity  $\mathbb{E}[\widehat{\ell}]_j = [f]_j[\lambda]_j$  we obtain

$$\begin{aligned}\square &= \sum_{0 \leq |j| \leq k_n^*} \omega_j \mathbb{E}[|\widehat{\ell}]_j / [\widehat{f}]_j - [\lambda]_j|^2 \mathbf{1}_{\Omega_j}] \\ &\leq 2 \sum_{0 \leq |j| \leq k_n^*} \omega_j \mathbb{E}[|\widehat{\ell}]_j / [\widehat{f}]_j - \mathbb{E}[\widehat{\ell}]_j / [\widehat{f}]_j|^2 \mathbf{1}_{\Omega_j}] + 2 \sum_{0 \leq |j| \leq k_n^*} \omega_j |[\lambda]_j|^2 \mathbb{E}[|[\widehat{f}]_j / [\widehat{f}]_j - 1|^2 \mathbf{1}_{\Omega_j}] \\ &=: 2\square_1 + 2\square_2.\end{aligned}$$

Using the estimate  $|a|^2 \leq 2|a-1|^2 + 2$  for  $a = [\widehat{\ell}]_j / [\widehat{f}]_j$ , the definition of  $\Omega_j$  and the independence of  $[\widehat{\ell}]_j$  and  $[\widehat{f}]_j$  we get

$$\begin{aligned}\square_1 &= \sum_{0 \leq |j| \leq k_n^*} \omega_j \mathbb{E} \left[ \left| \frac{[\widehat{\ell}]_j}{[\widehat{f}]_j} - \mathbb{E}[\widehat{\ell}]_j / [\widehat{f}]_j \right|^2 \cdot \left| \frac{[\widehat{f}]_j}{[\widehat{f}]_j} \right|^2 \mathbf{1}_{\Omega_j} \right] \\ &\leq 2 \sum_{0 \leq |j| \leq k_n^*} m\omega_j \frac{\text{Var}([\widehat{\ell}]_j) \text{Var}([\widehat{f}]_j)}{|[\widehat{f}]_j|^2} + 2 \sum_{0 \leq |j| \leq k_n^*} \omega_j \frac{\text{Var}([\widehat{f}]_j)}{|[\widehat{f}]_j|^2}.\end{aligned}$$

Applying statements a) and b) from Lemma A.1 together with  $f \in \mathcal{F}_\alpha^d$  yields

$$\square_1 \leq 4d \sum_{0 \leq |j| \leq k_n^*} \omega_j \frac{[\lambda]_0}{n\alpha_j}$$

which using that  $\gamma_j \geq 1$  (which holds due to Assumption A) implies

$$\square_1 \leq 4d\sqrt{r} \sum_{0 \leq |j| \leq k_n^*} \frac{\omega_j}{n\alpha_j} \leq 4d\sqrt{r} \cdot \Psi_n.$$

Now consider  $\square_2$ . Using the estimate  $|a|^2 \leq 2|a-1|^2 + 2$  for  $a = [\widehat{f}]_j / [\widehat{f}]_j$  and the definition of  $\Omega_j$  yields

$$\mathbb{E}[|[\widehat{f}]_j / [\widehat{f}]_j - 1|^2 \mathbf{1}_{\Omega_j}] \leq 2m \frac{\mathbb{E}[|[\widehat{f}]_j - [\widehat{f}]_j|^4]}{|[\widehat{f}]_j|^2} + 2 \frac{\text{Var}([\widehat{f}]_j)}{|[\widehat{f}]_j|^2}. \quad (\text{A.5})$$

Notice that Theorem 2.10 in [Pet95] implies the existence of a constant  $C > 0$  with  $\mathbb{E}[|[\widehat{f}]_j - [\widehat{f}]_j|^4] \leq C/m^2$ . Using this inequality in combination with assertion b) from Lemma A.1 and  $f \in \mathcal{F}_\alpha^d$  implies

$$\mathbb{E}[|[\widehat{f}]_j / [\widehat{f}]_j - 1|^2 \mathbf{1}_{\Omega_j}] \leq 2d(C+1)/(m\alpha_j). \quad (\text{A.6})$$

In addition,  $\mathbb{E}[|[\widehat{f}]_j / [\widehat{f}]_j - 1|^2 \mathbf{1}_{\Omega_j}] \leq m \text{Var}([\widehat{f}]_j) \leq 1$  which in combination with (A.6) implies

$$\square_2 \leq 2d(C+1) \sum_{0 \leq |j| \leq k_n^*} \omega_j |[\lambda]_j|^2 \min \left( 1, \frac{1}{m\alpha_j} \right).$$

Exploiting the fact that  $\lambda \in \Lambda_\gamma^r$  and the definition of  $\Phi_m$  in (3.2) we obtain

$$\square_2 \leq 2dr(C+1)(1 + \gamma_1/\omega_1) \cdot \Phi_m.$$

Putting together the estimates for  $\square_1$  and  $\square_2$  yields

$$\square \leq 8d\sqrt{r} \cdot \Psi_n + 4d(C+1)(1 + \gamma_1/\omega_1)r \cdot \Phi_m.$$

Upper bound for  $\Delta$ :  $\Delta$  can be decomposed as

$$\begin{aligned}\Delta &= \sum_{j \in \mathbb{Z}} \omega_j |\lambda_j|^2 \mathbb{E}(1 - \mathbb{1}_{\{0 \leq |j| \leq k_n^*\}} \cdot \mathbb{1}_{\Omega_j}) = \sum_{|j| > k_n^*} \omega_j |\lambda_j|^2 + \sum_{0 \leq |j| \leq k_n^*} \omega_j |\lambda_j|^2 \cdot \mathbb{P}(\Omega_j^c) \\ &= \Delta_1 + \Delta_2.\end{aligned}$$

$\lambda \in \Lambda_\gamma^r$  implies  $\Delta_1 \leq r\omega_{k_n^*}/\gamma_{k_n^*} \leq r \cdot \Psi_n$  and Lemma A.1 yields the estimate  $\Delta_2 \leq 4dr \cdot \Phi_m$  which together imply  $\Delta \leq r \cdot \Psi_n + 4dr \cdot \Phi_m$ . Putting the obtained estimates for  $\square$  and  $\Delta$  into (A.4) finishes the proof of the theorem.  $\square$

#### A.4. Auxiliary results for the proof of Theorem 3.6

LEMMA A.1 *With the notations introduced in the main part of the article, the following assertions hold:*

- a) i)  $\text{Var}(\widehat{[\ell]}_j) \leq [\lambda]_0/n$  under the Poisson model and
- ii)  $\text{Var}(\widehat{[\ell]}_j) \leq 2(|[\lambda]_j|^2 + [\lambda]_0)/n$  under the Cox model.
- b)  $\text{Var}(\widehat{[f]}_j) \leq 1/m$ ,
- c)  $\mathbb{P}(\Omega_j^c) = \mathbb{P}(|\widehat{[f]}_j|^2 < 1/m) \leq \min\{1, 4d/(m\alpha_j)\} \quad \forall f \in \mathcal{F}_\alpha^d$ .

*Proof.* The proof of statement i) in a) is given by the identity

$$\text{Var}(\widehat{[\ell]}_j) = \frac{1}{n} \text{Var} \left( \int_0^1 \mathbf{e}_j(t) dN_1(t) \right) = \frac{1}{n} \int_0^1 |\mathbf{e}_j(t)|^2 (\lambda \star f)(t) dt = \frac{1}{n} \cdot [\lambda]_0.$$

To prove ii), the identity  $\mathbb{E}[\widehat{[\ell]}_j] = [\lambda]_j[f]_j$  implies

$$\text{Var}(\widehat{[\ell]}_j) := \mathbb{E}[|\widehat{[\ell]}_j - \mathbb{E}[\widehat{[\ell]}_j]|^2] \leq 2\mathbb{E}[|\widetilde{[f]}_j[\lambda]_j - [f]_j[\lambda]_j|^2] + 2\mathbb{E}[|\xi_j|^2] =: 2V_1 + 2V_2$$

where  $V_1 \leq |[\lambda]_j|^2 \cdot \text{Var}(\widetilde{[f]}_j) \leq |[\lambda]_j|^2/n$ . Here, the estimate  $\text{Var}(\widetilde{[f]}_j) \leq 1/n$  is easily derived in analogy to the proof of part b). In order to bound  $V_2$  from above, notice

$$\begin{aligned}\mathbb{E}[|\xi_j|^2] &= \frac{1}{n} \mathbb{E} \left[ \mathbb{E} \left[ \left| \int_0^1 \mathbf{e}_j(-t) \{dN_1(t) - \lambda(t - \varepsilon_1 - \lfloor t - \varepsilon_1 \rfloor) dt\} \right|^2 \mid \varepsilon_1 \right] \right] \\ &= \frac{1}{n} \mathbb{E} \left[ \int_0^1 |\mathbf{e}_j(-t)|^2 \lambda(t - \varepsilon_1 - \lfloor t - \varepsilon_1 \rfloor) dt \right] \\ &= \frac{1}{n} \mathbb{E} \left[ \int_0^1 \lambda(t - \varepsilon_1 - \lfloor t - \varepsilon_1 \rfloor) dt \right] \\ &= [\lambda]_0/n.\end{aligned}$$

The assertion follows now by combining the obtained bounds for  $V_1$  and  $V_2$ .

For the proof of b), note that we have  $\text{Var}(\widehat{[f]}_j) = \frac{1}{m} \text{Var}(\mathbf{e}_j(-Y_1))$  and the assertion follows from the estimate

$$\text{Var}(\mathbf{e}_j(-Y_1)) = \mathbb{E}[|\mathbf{e}_j(-Y_1)|^2] - |\mathbb{E}[\mathbf{e}_j(-Y_1)]|^2 \leq \mathbb{E}[|\mathbf{e}_j(-Y_1)|^2] = 1.$$

For the proof of c), we consider two cases: if  $|[f]_j|^2 < 4/m$  we have  $1 < \frac{4d}{m\alpha_j}$  because  $f \in \mathcal{F}_\alpha^d$  and the statement is evident. Otherwise,  $|[f]_j|^2 \geq 4/m$  which implies

$$\mathbb{P}(|\widehat{[f]}_j|^2 < 1/m) \leq \mathbb{P}(|\widehat{[f]}_j|/|[f]_j| < 1/2) \leq \mathbb{P}(|\widehat{[f]}_j/[f]_j - 1| > 1/2).$$

Applying Chebyshev's inequality and exploiting the definition of  $\mathcal{F}_\alpha^d$  yields

$$\mathbb{P}(|\widehat{[f]}_j|^2 < 1/m) \leq 4/|[f]_j|^2 \cdot \text{Var}(\widehat{[f]}_j) \leq 4d/(m\alpha_j)$$

and statement c) follows.  $\square$

## B. Proofs of Section 4

### B.1. Proof of Theorem 4.1

Define the events  $\Xi_1 := \{\eta([\ell]_0 \vee 1) \leq \widehat{[\ell]}_0 \vee 1 \leq \eta^{-1}([\ell]_0 \vee 1)\}$  and

$$\Xi_2 := \left\{ \forall 0 \leq |j| \leq M_m^\alpha : |\widehat{[f]}_j^{-1} - [f]_j^{-1}| \leq \frac{1}{2|[f]_j|} \quad \text{and} \quad |\widehat{[f]}_j| \geq \frac{1}{m} \right\}.$$

The identity  $1 = \mathbb{1}_{\Xi_1 \cap \Xi_2} + \mathbb{1}_{\Xi_2^c} + \mathbb{1}_{\Xi_1^c \cap \Xi_2}$  provides the decomposition

$$\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2] = \underbrace{\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_1 \cap \Xi_2}]}_{=: \square_1} + \underbrace{\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_2^c}]}_{=: \square_2} + \underbrace{\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_1^c \cap \Xi_2}]}_{=: \square_3},$$

and we will establish uniform upper bounds over the ellipsoids  $\Lambda_\gamma^r$  and  $\mathcal{F}_\alpha^d$  for the three terms on the right-hand side separately.

*Uniform upper bound for  $\square_1$ :* Denote by  $\mathcal{S}_k$  the linear subspace of  $\mathbb{L}^2$  spanned by the functions  $\mathbf{e}_j(\cdot)$  for  $j \in \{-k, \dots, k\}$ . Since the identity  $\Upsilon(t) = \|t - \widehat{\lambda}_k\|_\omega^2 - \|\widehat{\lambda}_k\|_\omega^2$  holds for all  $t \in \mathcal{S}_k$ ,  $k \in \{0, \dots, n \wedge m\}$ , we obtain for all such  $k$  that  $\arg\min_{t \in \mathcal{S}_k} \Upsilon(t) = \widehat{\lambda}_k$ . Using this identity and the definition of  $\widehat{k}$  yields for all  $k \in \{0, \dots, K_{nm}^\alpha\}$  that

$$\Upsilon(\widehat{\lambda}_k) + \widetilde{\text{PEN}}_k^\sim \leq \Upsilon(\widehat{\lambda}_k) + \widetilde{\text{PEN}}_k \leq \Upsilon(\lambda_k) + \widetilde{\text{PEN}}_k$$

where  $\lambda_k := \sum_{0 \leq |j| \leq k} [\lambda]_j \mathbf{e}_j$  denotes the projection of  $\lambda$  on the subspace  $\mathcal{S}_k$ . Elementary computations imply

$$\|\widehat{\lambda}_k^\sim\|_\omega^2 \leq \|\lambda_k\|_\omega^2 + 2\Re\langle \widehat{\lambda}_{n \wedge m}, \widehat{\lambda}_k^\sim - \lambda_k \rangle_\omega + \widetilde{\text{PEN}}_k - \widetilde{\text{PEN}}_k^\sim \quad (\text{B.1})$$

for all  $k \in \{0, \dots, K_{nm}^\alpha\}$ . In addition to  $\lambda_k$  defined above, introduce the further abbreviations

$$\widetilde{\lambda}_k := \sum_{0 \leq |j| \leq k} \frac{\widehat{[\ell]}_j}{[f]_j} \mathbf{e}_j \quad \text{and} \quad \check{\lambda}_k := \sum_{0 \leq |j| \leq k} \frac{[\ell]_j}{[f]_j} \mathbf{1}_{\Omega_j} \mathbf{e}_j,$$

as well as

$$\Theta_k := \widehat{\lambda}_k - \check{\lambda}_k - \widetilde{\lambda}_k + \lambda_k, \quad \widetilde{\Theta}_k := \widetilde{\lambda}_k - \lambda_k, \quad \text{and} \quad \check{\Theta}_k := \check{\lambda}_k - \lambda_k.$$

Using these abbreviations and the identity  $\widehat{\lambda}_{n \wedge m} - \lambda_{n \wedge m} = \Theta_{n \wedge m} + \widetilde{\Theta}_{n \wedge m} + \check{\Theta}_{n \wedge m}$ , we deduce from (B.1) that

$$\begin{aligned} \|\widehat{\lambda}_k^\sim - \lambda\|_\omega^2 &\leq \|\lambda - \lambda_k\|_\omega^2 + \widetilde{\text{PEN}}_k - \widetilde{\text{PEN}}_k^\sim + 2\Re\langle \widetilde{\Theta}_{n \wedge m}, \widehat{\lambda}_k^\sim - \lambda_k \rangle_\omega \\ &\quad + 2\Re\langle \Theta_{n \wedge m}, \widehat{\lambda}_k^\sim - \lambda_k \rangle_\omega + 2\Re\langle \check{\Theta}_{n \wedge m}, \widehat{\lambda}_k^\sim - \lambda_k \rangle_\omega \end{aligned} \quad (\text{B.2})$$

for all  $k \in \{0, \dots, K_{nm}^\alpha\}$ . Define  $\mathcal{B}_k := \{\lambda \in \mathcal{S}_k : \|\lambda\|_\omega \leq 1\}$ . For every  $\tau > 0$  and  $t \in \mathcal{S}_k$ , the estimate  $2uv \leq \tau u^2 + \tau^{-1}v^2$  implies

$$2|\langle h, t \rangle_\omega| \leq 2\|t\|_\omega \sup_{t \in \mathcal{B}_k} |\langle h, t \rangle_\omega| \leq \tau \|t\|_\omega^2 + \frac{1}{\tau} \sup_{t \in \mathcal{B}_k} |\langle h, t \rangle_\omega|^2.$$



Because  $\widehat{\lambda}_k - \lambda_k \in \mathcal{S}_{k \vee k}$ , combining the last estimate with (B.2) we get

$$\begin{aligned} \|\widehat{\lambda}_k - \lambda\|_\omega^2 &\leq \|\lambda - \lambda_k\|_\omega^2 + 3\tau \|\widehat{\lambda}_k - \lambda_k\|_\omega^2 + \widetilde{\text{PEN}}_k - \widetilde{\text{PEN}}_k + \\ &\quad + \tau^{-1} \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 + \tau^{-1} \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \Theta_{n \wedge m}, t \rangle_\omega|^2 + \tau^{-1} \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2. \end{aligned}$$

Note that  $\|\widehat{\lambda}_k - \lambda_k\|_\omega^2 \leq 2\|\widehat{\lambda}_k - \lambda\|_\omega^2 + 2\|\lambda_k - \lambda\|_\omega^2$  and  $\|\lambda - \lambda_k\|_\omega^2 \leq r\omega_k\gamma_k^{-1}$  for all  $\lambda \in \Lambda_\gamma^r$  since  $\omega\gamma^{-1}$  is non-increasing due to Assumption A. Specializing with  $\tau = 1/8$ , we obtain

$$\begin{aligned} \|\widehat{\lambda}_k - \lambda\|_\omega^2 &\leq 7r\omega_k\gamma_k^{-1} + 4\widetilde{\text{PEN}}_k - 4\widetilde{\text{PEN}}_k + 32 \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 \\ &\quad + 32 \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \Theta_{n \wedge m}, t \rangle_\omega|^2 + 32 \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2. \end{aligned} \quad (\text{B.3})$$

Combining the facts that  $\mathbf{1}_{\Omega_j} \mathbf{1}_{\Xi_2} = \mathbf{1}_{\Xi_2}$  for  $0 \leq |j| \leq M_m^\alpha$  and  $K_{nm}^\alpha \leq M_m^\alpha$  by definition, we obtain for all  $j \in \{-K_{nm}^\alpha, \dots, K_{nm}^\alpha\}$  the estimate

$$|[f]_j / [\widehat{f}]_j| \mathbf{1}_{\Omega_j} - 1|^2 \mathbf{1}_{\Xi_2} = |[f]_j|^2 |1/[\widehat{f}]_j - 1/[f]_j|^2 \mathbf{1}_{\Xi_2} \leq 1/4.$$

Hence,  $\sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}, t \rangle_\omega|^2 \mathbf{1}_{\Xi_2} \leq \frac{1}{4} \sup_{t \in \mathcal{B}_k} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2$  for all  $0 \leq k \leq K_{nm}^\alpha$ . Thus, from (B.3) we obtain

$$\begin{aligned} \|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbf{1}_{\Xi_1 \cap \Xi_2} &\leq 7r\omega_k\gamma_k^{-1} + 40 \left( \sup_{t \in \mathcal{B}_{k \vee k}} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 - \frac{33d([\ell]_0 \vee 1)\delta_{k \vee k}^\alpha}{8n} \right) + \\ &\quad + (165d([\ell]_0 \vee 1)\delta_{k \vee k}^\alpha/n + 4\widetilde{\text{PEN}}_k - 4\widetilde{\text{PEN}}_k) \mathbf{1}_{\Xi_1 \cap \Xi_2} + 32 \sup_{t \in \mathcal{B}_{K_{nm}^\alpha}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2. \end{aligned}$$

Exploiting the definition of both the penalty  $\widetilde{\text{PEN}}$  and the event  $\Xi_1$ , we obtain

$$\begin{aligned} \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbf{1}_{\Xi_1 \cap \Xi_2}] &\leq C(d, r) \min_{0 \leq k \leq K_{nm}^\alpha} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\delta_k^\alpha}{n} \right\} \\ &\quad + 40 \sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 - \frac{33([\ell]_0 \vee 1)d\delta_k^\alpha}{8n} \right)_+ \right] \\ &\quad + 32 \mathbb{E} \left[ \sup_{t \in \mathcal{B}_{K_{nm}^\alpha}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2 \right]. \end{aligned} \quad (\text{B.4})$$

Applying Lemma B.2 with  $\delta_k^* = d\delta_k^\alpha$  and  $\Delta_k^* = d\Delta_k^\alpha$  yields

$$\begin{aligned} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 - \frac{33d([\ell]_0 \vee 1)\delta_k^\alpha}{8n} \right)_+ \right] &\leq K_1 \left[ \frac{d\|f\|\|\lambda\|\Delta_k^\alpha}{n} \exp \left( -K_2 \frac{\delta_k^\alpha}{\|f\|^2 \|\lambda\|^2 \Delta_k^\alpha} \right) \right. \\ &\quad \left. + \frac{d\delta_k^\alpha}{n^2} \exp(-K_3\sqrt{n}) \right]. \end{aligned}$$

Using statement a) of Lemma B.1 and the fact that  $K_{nm}^\alpha \leq n$  by definition, we obtain that

$$\begin{aligned} \sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 - \frac{33d([\ell]_0 \vee 1)\delta_k^\alpha}{8n} \right)_+ \right] \\ \lesssim \frac{d^{3/2}\sqrt{r\rho}}{n} \sum_{k=0}^{\infty} \Delta_k^\alpha \exp \left( -\frac{2K_2k}{\sqrt{dr\rho}} \cdot \frac{\log(\Delta_k^\alpha \vee (k+3))}{\log(k+3)} \right) + \exp(-K_3\sqrt{n}), \end{aligned}$$

where the last estimate is due to the fact that  $\|f\|^2 \leq d\rho$  for all  $f \in \mathcal{F}_\alpha^d$  and  $\|\lambda\|^2 \leq r$  for all  $\lambda \in \Lambda_\gamma^r$ . Note that we have

$$\sum_{k=0}^{\infty} \Delta_k^\alpha \exp\left(-\frac{2K_2k}{\sqrt{dr\rho}} \cdot \frac{\log(\Delta_k^\alpha \vee (k+3))}{\log(k+3)}\right) \leq C < \infty$$

with a numerical constant  $C$  which implies

$$\sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \tilde{\Theta}_{n \wedge m}, t \rangle_\omega| - \frac{33d([\ell]_0 \vee 1)\delta_k^\alpha}{8n} \right)_+ \right] \lesssim \frac{1}{n}.$$

The last term in (B.4) is bounded by means of Lemma B.3 which immediately yields

$$\mathbb{E} \left[ \sup_{t \in \mathcal{B}_{K_{nm}^\alpha}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2 \right] \lesssim \Phi_m.$$

Combining the preceeding estimates, which hold uniformly for all  $\lambda \in \Lambda_\gamma^r$  and  $f \in \mathcal{F}_\alpha^d$ , we conclude from equation (B.4) that

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E} \left[ \|\hat{\lambda}_k^\alpha - \lambda\|_\omega^2 \mathbf{1}_{\Xi_1 \cap \Xi_2} \right] \lesssim \min_{0 \leq k \leq K_{nm}^\alpha} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\delta_k^\alpha}{n} \right\} + \Phi_m + \frac{1}{n}.$$

*Uniform upper bound for  $\square_2$ :* Define  $\check{\lambda}_k := \sum_{0 \leq |j| \leq k} [\lambda]_j \mathbf{1}_{\Omega_j} \mathbf{e}_j$ . Note that  $\|\hat{\lambda}_k - \check{\lambda}_k\|_\omega^2 \leq \|\hat{\lambda}_{k'} - \check{\lambda}_{k'}\|_\omega^2$  for  $k \leq k'$  and  $\|\check{\lambda}_k - \lambda\|_\omega^2 \leq \|\lambda\|_\omega^2$  for all  $k \in \mathbb{N}_0$ . Consequently, since  $k \in \{0, \dots, K_{nm}^\alpha\}$ , we obtain the estimate

$$\begin{aligned} \mathbb{E}[\|\hat{\lambda}_k^\alpha - \lambda\|_\omega^2 \mathbf{1}_{\Xi_2^c}] &\leq 2\mathbb{E}[\|\hat{\lambda}_k^\alpha - \check{\lambda}_k^\alpha\|_\omega^2 \mathbf{1}_{\Xi_2^c}] + 2\mathbb{E}[\|\check{\lambda}_k^\alpha - \lambda\|_\omega^2 \mathbf{1}_{\Xi_2^c}] \\ &\leq 2\mathbb{E}[\|\hat{\lambda}_{K_{nm}^\alpha}^\alpha - \check{\lambda}_{K_{nm}^\alpha}^\alpha\|_\omega^2 \mathbf{1}_{\Xi_2^c}] + 2\|\lambda\|_\omega^2 \mathbb{P}(\Xi_2^c), \end{aligned}$$

and due to Assumption A and Lemma B.5 it is easily seen that  $\|\lambda\|_\omega^2 \cdot \mathbb{P}(\Xi_2^c) \lesssim m^{-4}$ . Using the definition of  $\Omega_j$ , we further obtain

$$\begin{aligned} \mathbb{E}[\|\hat{\lambda}_{K_{nm}^\alpha}^\alpha - \check{\lambda}_{K_{nm}^\alpha}^\alpha\|_\omega^2 \mathbf{1}_{\Xi_2^c}] &\leq 2m \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \{ \mathbb{E}[|\widehat{[\ell]}_j - [\ell]_j|^2 \mathbf{1}_{\Xi_2^c}] + \mathbb{E}[|[\widehat{f}]_j[\lambda]_j - [\widehat{f}]_j[\lambda]_j|^2 \mathbf{1}_{\Xi_2^c}] \} \\ &\leq 2m \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j (\mathbb{E}[|\widehat{[\ell]}_j - [\ell]_j|^4])^{1/2} \mathbb{P}(\Xi_2^c)^{1/2} \\ &\quad + 2m \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j |[\lambda]_j|^2 (\mathbb{E}[|\widehat{[f]}_j - [f]_j|^4])^{1/2} \mathbb{P}(\Xi_2^c)^{1/2} \\ &\lesssim m \mathbb{P}(\Xi_2^c)^{1/2} \sum_{0 \leq |j| \leq K_{nm}^\alpha} \frac{\omega_j}{n} + \mathbb{P}(\Xi_2^c)^{1/2} \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j |[\lambda]_j|^2, \end{aligned} \quad (\text{B.5})$$

where the last estimate follows by applying Theorem 2.10 from [Pet95] with  $p = 4$  two times. If  $K_{nm}^\alpha = 0$ , Lemma B.5 implies

$$\mathbb{E}[\|\hat{\lambda}_{K_{nm}^\alpha}^\alpha - \check{\lambda}_{K_{nm}^\alpha}^\alpha\|_\omega^2 \mathbf{1}_{\Xi_2^c}] \lesssim \frac{1}{nm} + \frac{1}{m^2}.$$

Otherwise, if  $K_{nm}^\alpha > 0$ , we exploit  $\omega_j \leq \omega_j^+ \alpha_j^{-1}$ ,  $K_{nm}^\alpha \leq N_n^\alpha$  and the definition of  $N_n^\alpha$  to bound the first term in (B.5). The second term in (B.5) can be bounded from above by noting that

$\omega_j \leq \gamma_j$  thanks to Assumption A, and we obtain

$$\mathbb{E}[\|\widehat{\lambda}_{K_{nm}^\alpha} - \check{\lambda}_{K_{nm}^\alpha}\|_\omega^2 \mathbf{1}_{\Xi_2^c}] \lesssim m \mathbb{P}(\Xi_2^c)^{1/2} \left( \sum_{0 \leq |j| \leq N_n^\alpha} \frac{1}{2|j|+1} \right) \frac{1}{\log(n+3)} + \mathbb{P}(\Xi_2^c)^{1/2}.$$

Thanks to the logarithmic increase of the harmonic series,  $N_n^\alpha \leq n$  and Lemma B.5, the last estimate implies

$$\mathbb{E}[\|\widehat{\lambda}_{K_{nm}^\alpha} - \check{\lambda}_{K_{nm}^\alpha}\|_\omega^2 \mathbf{1}_{\Xi_2^c}] \lesssim \frac{1}{m} + \frac{1}{m^2},$$

if  $K_{nm}^\alpha > 0$ , and thus

$$\mathbb{E}[\|\widehat{\lambda}_{K_{nm}^\alpha} - \check{\lambda}_{K_{nm}^\alpha}\|_\omega^2 \mathbf{1}_{\Xi_2^c}] \lesssim \frac{1}{m} + \frac{1}{m^2},$$

independent of the actual value of  $K_{nm}^\alpha$ . Using the obtained estimates, we conclude

$$\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbf{1}_{\Xi_2^c}] \lesssim \frac{1}{m}.$$

*Uniform upper bound for  $\square_3$ :* In order to find a uniform upper bound for  $\square_3$ , first recall the definition  $\lambda_k := \sum_{0 \leq |j| \leq k} [\lambda]_j \mathbf{1}_{\Omega_j} \mathbf{e}_j$  and consider the estimate

$$\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] \leq 2\mathbb{E}[\|\widehat{\lambda}_k - \check{\lambda}_k\|_\omega^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] + 2\mathbb{E}[\|\check{\lambda}_k - \lambda\|_\omega^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}]. \quad (\text{B.6})$$

Using the estimate  $\|\check{\lambda}_k - \lambda\|_\omega^2 \leq \|\lambda\|_\omega^2$ , we obtain for  $\lambda \in \Lambda_\gamma^r$  by means of Lemma B.4 that

$$\mathbb{E}[\|\check{\lambda}_k - \lambda\|_\omega^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] \leq r \mathbb{P}(\Xi_1^c) \lesssim \frac{1}{n}$$

which controls the second term on the right-hand side of (B.6). We now bound the first term on the right-hand side of (B.6). If  $K_{nm}^\alpha = 0$ , we have  $\check{k} = 0$ , and by means of the Cauchy-Schwarz inequality and Theorem 2.10 from [Pet95] it is easily seen that

$$\mathbb{E}[\|\widehat{\lambda}_k - \check{\lambda}_k\|_\omega^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] \lesssim \frac{1}{n}.$$

Otherwise,  $K_{nm}^\alpha > 0$ , and we need the following further estimate, which is easily verified:

$$\begin{aligned} \mathbb{E}[\|\widehat{\lambda}_k - \check{\lambda}_k\|_\omega^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] &\leq 3 \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|[\ell]_j / [\widehat{f}]_j - [\ell]_j / [f]_j|^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] \\ &\quad + 3 \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|\widehat{[\ell]}_j - [\ell]_j|^2 / |[f]_j|^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] \\ &\quad + 3 \sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|\widehat{[\ell]}_j - [\ell]_j|^2 \cdot |1 / [\widehat{f}]_j - 1 / [f]_j|^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}]. \end{aligned} \quad (\text{B.7})$$

We start by bounding the first term on the right-hand side of (B.7). Using the definition of  $\Xi_2$  and  $\omega_j \leq \gamma_j$ , we obtain for all  $\lambda \in \Lambda_\gamma^r$  that

$$\sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|[\ell]_j / [\widehat{f}]_j - [\ell]_j / [f]_j|^2 \mathbf{1}_{\Xi_1^c \cap \Xi_2}] \leq \frac{r}{4} \cdot \mathbb{P}(\Xi_1^c) \lesssim \frac{1}{n}.$$

Since  $|[f]_j|^{-2} \leq d\alpha_j$  for  $f \in \mathcal{F}_\alpha^d$ , the Cauchy-Schwarz inequality in combination with Theo-

rem 2.10 from [Pet95] implies for the second term on the right-hand side of (B.7) that

$$\sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|\widehat{\ell}]_j - [\ell]_j|^2 / |[f]_j|^2 \mathbb{1}_{\Xi_1^\complement \cap \Xi_2}] \lesssim \mathbb{P}(\Xi_1^\complement)^{1/2} \sum_{0 \leq |j| \leq K_{nm}^\alpha} \frac{\omega_j^+}{n\alpha_j}.$$

We exploit the definition of  $N_n^\alpha$  together with  $K_{nm}^\alpha \leq N_n^\alpha$  to obtain

$$\sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|\widehat{\ell}]_j - [\ell]_j|^2 / |[f]_j|^2 \mathbb{1}_{\Xi_1^\complement \cap \Xi_2}] \lesssim \frac{\mathbb{P}(\Xi_1^\complement)^{1/2}}{\log(n+3)} \sum_{0 \leq |j| \leq N_n^\alpha} \frac{1}{2|j|+1},$$

from which by the logarithmic increase of the harmonic series and Lemma B.4 we conclude that

$$\sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|\widehat{\ell}]_j - [\ell]_j|^2 / |[f]_j|^2 \mathbb{1}_{\Xi_1^\complement \cap \Xi_2}] \lesssim \frac{1}{n},$$

independent of the actual value of  $K_{nm}^\alpha$ . Finally, the third and last term on the right-hand side of (B.7) can be bounded from above the same way after exploiting the definition of  $\Xi_2$ , and we obtain

$$\sum_{0 \leq |j| \leq K_{nm}^\alpha} \omega_j \mathbb{E}[|\widehat{\ell}]_j - [\ell]_j|^2 \cdot |1/[\widehat{f}]_j - 1/[f]_j|^2 \mathbb{1}_{\Xi_1^\complement \cap \Xi_2}] \lesssim \frac{1}{n}.$$

Putting together the derived estimates, we obtain

$$\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|^2 \mathbb{1}_{\Xi_1^\complement \cap \Xi_2}] \lesssim \frac{1}{n}.$$

Finally, the statement of the theorem follows by combining the obtained uniform upper bounds for  $\square_1$ ,  $\square_2$ , and  $\square_3$ .  $\square$

## B.2. Proof of Theorem 4.2

Consider the event

$$\Xi_3 := \{N_n^{\alpha-} \wedge M_m^{\alpha-} \leq \widehat{K}_{nm} \leq N_n^{\alpha+} \wedge M_m^{\alpha+}\}$$

in addition to the event  $\Xi_1$  introduced in the proof of Theorem 4.1 and the slightly redefined event  $\Xi_2$  defined as

$$\Xi_2 := \{\forall 0 \leq |j| \leq M_m^{\alpha+} : |1/[\widehat{f}]_j - 1/[f]_j| \leq 1/(2|[f]_j|) \text{ and } |[\widehat{f}]_j| \geq 1/m\}.$$

Defining  $\Xi := \Xi_1 \cap \Xi_2 \cap \Xi_3$ , the identity  $1 = \mathbb{1}_\Xi + \mathbb{1}_{\Xi_2^\complement} + \mathbb{1}_{\Xi_1^\complement \cap \Xi_2} + \mathbb{1}_{\Xi_1 \cap \Xi_2 \cap \Xi_3^\complement}$  motivates the decomposition

$$\begin{aligned} \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2] &= \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_\Xi] + \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_2^\complement}] \\ &\quad + \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_1^\complement \cap \Xi_2}] + \mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_1 \cap \Xi_2 \cap \Xi_3^\complement}] \\ &=: \square_1 + \square_2 + \square_3 + \square_4, \end{aligned}$$

and we establish uniform upper risk bounds for the four terms on the right-hand side separately. *Uniform upper bound for  $\square_1$ :* On  $\Xi$  we have the estimate  $\frac{1}{4}\Delta_k \leq \widehat{\Delta}_k \leq \frac{9}{4}\Delta_k$ , and thus

$$\frac{1}{4} [\Delta_k \vee (k+4)] \leq \widehat{\Delta}_k \vee (k+4) \leq \frac{9}{4} [\Delta_k \vee (k+4)]$$

for all  $k \in \{0, \dots, M_m^{\alpha+}\}$ . This last estimate implies

$$\begin{aligned} \frac{2k+1}{4} \Delta_k \frac{\log(\Delta_k \vee (k+4))}{\log(k+4)} \left(1 - \frac{\log 4}{\log(k+4)} \frac{\log(k+4)}{\log(\Delta_k \vee (k+4))}\right) &\leq \widehat{\delta}_k \\ &\leq \frac{9(2k+1)}{4} \Delta_k \frac{\log(\Delta_k \vee (k+4))}{\log(k+4)} \left(1 + \frac{\log(9/4)}{\log(k+4)} \frac{\log(k+4)}{\log(\Delta_k \vee (k+4))}\right), \end{aligned}$$

from which we conclude  $\frac{3}{100} \cdot \delta_k \leq \widehat{\delta}_k \leq \frac{17}{5} \cdot \delta_k$ . Putting  $\text{PEN}_k := \frac{165}{4} \eta^{-1}([\widehat{\ell}]_0 \vee 1) \cdot \frac{\delta_k}{n}$ , we observe that on  $\Xi_2$  the estimate

$$\text{PEN}_k \leq \widehat{\text{PEN}}_k \leq \frac{340}{3} \text{PEN}_k$$

holds for all  $k \in \{0, \dots, M_m^{\alpha+}\}$ . Note that on  $\Xi$  we have  $\widehat{k} \leq M_m^{\alpha+}$  which implies

$$(\text{PEN}_{k \vee \widehat{k}} + \widehat{\text{PEN}}_k - \widehat{\text{PEN}}_{\widehat{k}}) \mathbf{1}_{\Xi} \leq (\text{PEN}_k + \text{PEN}_{\widehat{k}} + \widehat{\text{PEN}}_k - \widehat{\text{PEN}}_{\widehat{k}}) \mathbf{1}_{\Xi} \leq \frac{343}{3} \text{PEN}_k \mathbf{1}_{\Xi}. \quad (\text{B.8})$$

Now, we can proceed by mimicking the derivation of (B.4) in the proof of Theorem 4.1. More precisely, replacing the penalty term  $\widetilde{\text{PEN}}_k$  used in that proof by  $\widehat{\text{PEN}}_k$ , using the definition of  $\text{PEN}_k$  above and (B.8), we obtain

$$\begin{aligned} \mathbb{E}[\|\widehat{\lambda}_{\widehat{k}} - \lambda\|_{\omega}^2 \mathbf{1}_{\Xi}] &\leq 7r\omega_k \gamma_k^{-1} + 40 \sum_{k=0}^{N_n^{\alpha+}} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_{\omega}|^2 - \frac{33([\ell]_0 \vee 1)\delta_k}{8n} \right)_+ \right] \\ &\quad + 32\mathbb{E} \left[ \sup_{t \in \mathcal{B}_{K_{nm}^{\alpha+}}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_{\omega}|^2 \right] + 4\mathbb{E}[(\text{PEN}_{k \vee \widehat{k}} + \widehat{\text{PEN}}_k - \widehat{\text{PEN}}_{\widehat{k}}) \mathbf{1}_{\Xi}] \\ &\leq 7r\omega_k \gamma_k^{-1} + 40 \sum_{k=0}^{N_n^{\alpha+}} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_{\omega}|^2 - \frac{33([\ell]_0 \vee 1)\delta_k}{8n} \right)_+ \right] \\ &\quad + 32\mathbb{E} \left[ \sup_{t \in \mathcal{B}_{K_{nm}^{\alpha+}}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_{\omega}|^2 \right] + \frac{1372}{3} \text{PEN}_k. \end{aligned}$$

As in the proof of Theorem 4.1, the second and the third term are bounded applying Lemmata B.2 (with  $\delta_k^* := \delta_k$  and  $\Delta_k^* := \Delta$ ) and B.3, respectively. Hence, by means of an obvious adaption of statement a) in Lemma B.1 (with  $N_n^{\alpha}$  replaced by  $N_n^{\alpha+}$ ) and the estimates

$$\Delta_k \leq d\Delta_k^{\alpha}, \quad \delta_k \leq d\zeta_d \delta_k^{\alpha}, \quad \frac{\delta_k}{\Delta_k} \geq 2k\zeta_d^{-1} \frac{\log(\Delta_k^{\alpha} \vee (k+4))}{\log(k+4)}$$

with  $\zeta_d = \log(4d)/\log(4)$ , we obtain in analogy to the way of proceeding in the proof of Theorem 4.1 that

$$\sup_{\lambda \in \Lambda_{\gamma}^r} \sup_{f \in \mathcal{F}_{\alpha}^d} \mathbb{E}[\|\widehat{\lambda}_{\widehat{k}} - \lambda\|_{\omega}^2 \mathbf{1}_{\Xi}] \lesssim \min_{0 \leq k \leq K_{nm}^{\alpha-}} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\delta_k^{\alpha}}{n} \right\} + \Phi_m + \frac{1}{n}. \quad (\text{B.9})$$

*Upper bound for  $\square_2$ :* The uniform upper bound for  $\square_2$  can be derived in analogy to the bound for  $\square_2$  in the proof of Theorem 4.1 using Assumption B instead of statement b) from Lemma B.1 in the proof of Lemma B.5. Hence, we obtain

$$\sup_{\lambda \in \Lambda_{\gamma}^r} \sup_{f \in \mathcal{F}_{\alpha}^d} \mathbb{E}[\|\widehat{\lambda}_{\widehat{k}} - \lambda\|_{\omega}^2 \mathbf{1}_{\Xi_2}] \lesssim \frac{1}{m}. \quad (\text{B.10})$$

*Upper bound for  $\square_3$ :* The term  $\square_3$  can also be bounded analogously to the bound established for  $\square_3$  in the proof of Theorem 4.1 (here, we do not have to exploit the additional Assumption B),

and we get

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\hat{\lambda}_k - \lambda\|^2 \mathbb{1}_{\Xi_1^c \cap \Xi_2}] \lesssim \frac{1}{n}. \quad (\text{B.11})$$

*Upper bound for  $\square_4$ :* To find a uniform upper bound for the term  $\square_4$ , one can use exactly the same decompositions as in the proof of the uniform upper bound for  $\square_3$  in Theorem 4.1 by replacing the probability of  $\Xi_1^c$  with the one of  $\Xi_3^c$ . Doing this, we obtain by means of Lemma B.6 that

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\hat{\lambda}_k - \lambda\|^2 \mathbb{1}_{\Xi_1 \cap \Xi_2 \cap \Xi_3^c}] \lesssim \frac{1}{m}. \quad (\text{B.12})$$

The result of the theorem now follows by combining (B.9), (B.10), (B.11) and (B.12).  $\square$

### B.3. Auxiliary results

LEMMA B.1 *Let Assumption A hold. Then the following assertions hold true.*

- a)  $\delta_j^\alpha/n \leq 1$  for all  $n \in \mathbb{N}$  and  $j \in \{0, \dots, N_n^\alpha\}$ ,
- b)  $\exp(-m\alpha_{M_m^\alpha}/(128d)) \leq C(d)m^{-5}$  for all  $m \in \mathbb{N}$ , and
- c)  $\min_{1 \leq j \leq M_m^\alpha} |[f]_j|^2 \geq 2m^{-1}$  for all  $m \in \mathbb{N}$ .

*Proof.* a) In case  $N_n^\alpha = 0$ , we have  $\delta_{N_n^\alpha}^\alpha = 1$  and there is nothing to show. Otherwise  $0 < N_n^\alpha \leq n$ , and by definition of  $N_n^\alpha$  we have  $(2j+1)\Delta_j^\alpha \leq n/\log(n+3)$  for  $0 \leq j \leq N_n^\alpha$  which by the definition of  $\delta_j^\alpha$  implies that

$$\delta_j^\alpha \leq \frac{n}{\log(n+3)} \cdot \frac{\log(n/((2j+1)\log(n+3)) \vee (j+3))}{\log(j+3)}.$$

We consider two cases: In the first case,  $n/((2j+1)\log(n+3)) \vee (j+3) = j+3$ . Then  $n \geq 1$  directly implies the estimate  $\delta_j^\alpha \leq n$ . In the second case, we have  $n/((2j+1)\log(n+3)) \vee (j+3) = n/((2j+1)\log(n+3))$  and therefrom

$$\delta_j^\alpha \leq n \log(n)/(\log(n+3)\log(j+3)) \leq n,$$

and thus  $\delta_j^\alpha \leq n$  in both cases. Division by  $n$  yields the assertion of the lemma.

b) Note that, due to Assumption A, we have  $M_m^\alpha > 0$  for all sufficiently large  $m$  and that it is sufficient to show the desired inequality for such values of  $m$ . By the definition of  $M_m^\alpha$ , we have  $\alpha_{M_m^\alpha} \geq 640dm^{-1} \cdot \log(m+1)$  which implies

$$\exp(-m\alpha_{M_m^\alpha}/(128d)) \leq \exp(-5 \log m) = m^{-5},$$

and the assertion follows.

c) Take note of the observation that

$$\min_{1 \leq j \leq M_m^\alpha} |[f]_j|^2 \geq \min_{1 \leq j \leq M_m^\alpha} \frac{\alpha_j}{d} = \frac{\alpha_{M_m^\alpha}}{d} \geq 640m^{-1} \cdot \log(m+1)$$

and  $640m^{-1} \cdot \log(m+1) \geq 2m^{-1}$  for all  $m \geq 1$ .  $\square$

LEMMA B.2 *Let  $(\delta_k^*)_{k \in \mathbb{N}_0}$  and  $(\Delta_k^*)_{k \in \mathbb{N}_0}$  be sequences such that for all  $k \geq 1$ ,*

$$\delta_k^* \geq \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2} \quad \text{and} \quad \Delta_k^* \geq \max_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2}.$$

Then, for any  $k \in \{1, \dots, n \wedge m\}$ , we have

$$\begin{aligned} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \check{\Theta}_{n \wedge m}, t \rangle|^2 - \frac{33\delta_k^*([\ell]_0 \vee 1)}{8n} \right)_+ \right] \\ \leq K_1 \left\{ \frac{\|f\| \|\lambda\| \Delta_k^*}{n} \exp \left( -K_2 \cdot \frac{\delta_k^*}{\|f\| \|\lambda\| \Delta_k^*} \right) + \frac{\delta_k^*}{n^2} \exp(-K_3 \sqrt{n}) \right\}, \end{aligned}$$

with positive numerical constants  $K_1$ ,  $K_2$ , and  $K_3$ .

*Proof.* The proof is a combination of the proofs of Lemma A.1 in [Kro16] (which deals with the case  $\omega \equiv 1$ ) and Lemma A.4 in [JS13] (where general sequences  $\omega$  are considered in the framework with random variables instead of point processes). More precisely, one can apply Proposition C.1 in [Kro16] with  $c(\varepsilon)$  from that statement replaced with  $c(\varepsilon) = 4(1 + 2\varepsilon)$  (this makes the proposition applicable also for complex-valued functions),  $M_1^2 = \delta_k^*$ ,  $H^2 = \frac{\delta_k^*}{n}([\ell]_0 \vee 1)$ ,  $v := \|\lambda\| \|f\| \Delta_k^*([\ell]_0 \vee 1)$  and setting  $\varepsilon = \frac{1}{64}$ .  $\square$

LEMMA B.3 *Let  $m \in \mathbb{N}$  and  $k \in \mathbb{N}_0$ . Then*

$$\sup_{\lambda \in \Lambda_\gamma^r} \mathbb{E} \left[ \sup_{t \in \mathcal{B}_k} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2 \right] \leq C(d, r) \cdot \Phi_m.$$

*Proof.* Note that  $\lambda \in \Lambda_\gamma^r$  implies

$$\mathbb{E}[\sup_{t \in \mathcal{B}_k} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2] \leq r \sup_{-k \leq j \leq k} \omega_j \gamma_j^{-1} \mathbb{E}[|[f]_j / \widehat{[f]}_j| \cdot \mathbf{1}_{\Omega_j} - 1|^2]$$

Thus, recalling the definition of  $\Phi_m$  in (3.2), it suffices to show that

$$\mathbb{E}[|[f]_j / \widehat{[f]}_j| \cdot \mathbf{1}_{\Omega_j} - 1|^2] \leq C(d, r) \min\{1, 1/(m\alpha_j)\},$$

which can be realised by means of the identity

$$\mathbb{E}[|[f]_j / \widehat{[f]}_j| \cdot \mathbf{1}_{\Omega_j} - 1|^2] = \mathbb{E}[|[f]_j / \widehat{[f]}_j| \mathbf{1}_{\Omega_j} - 1|^2 \cdot \mathbf{1}_{\Omega_j}] + \mathbb{P}(\Omega_j^c) =: \square + \triangle.$$

The bound  $\square \leq C(d, r) \min\{1, 1/(m\alpha_j)\}$  was already derived in the proof of Theorem 3.6. For  $\triangle$ , the corresponding upper bound can be obtained from statement c) of Lemma A.1.  $\square$

LEMMA B.4 *Let Assumption A hold and consider the event  $\Xi_1$  defined in Theorem 4.1. Then, for any  $n \in \mathbb{N}$ ,  $\mathbb{P}(\Xi_1^c) \leq 2 \exp(-Cn)$  with a numerical constant  $C = C(\eta) > 0$ .*

*Proof.* Note that

$$\mathbb{P}(\Xi_1^c) = \mathbb{P}(\widehat{[\ell]}_0 \vee 1 < \eta([\ell]_0 \vee 1)) + \mathbb{P}(\widehat{[\ell]}_0 \vee 1 > \eta^{-1}([\ell]_0 \vee 1)),$$

and the two terms on the right-hand side can be bounded by Chernoff bounds for Poisson distributed random variables (see [MU05], Theorem 5.4). More precisely, we have

$$\begin{aligned} \mathbb{P}(\widehat{[\ell]}_0 \vee 1 < \eta([\ell]_0 \vee 1)) &\leq \exp(-\omega_1(\eta)n), \quad \text{and} \\ \mathbb{P}(\widehat{[\ell]}_0 \vee 1 > \eta^{-1}([\ell]_0 \vee 1)) &\leq \exp(-\omega_2(\eta)n) \end{aligned}$$

with  $\omega_1(\eta) = 1 - \eta + \eta \log \eta > 0$  and  $\omega_2(\eta) = 1 - \eta^{-1} - \eta^{-1} \log \eta > 0$  for all  $\eta \in (0, 1)$ .  $\square$

LEMMA B.5 *Let Assumption A hold and consider the event  $\Xi_2$  defined in the proof of Theorem 4.1. Then, for any  $m \in \mathbb{N}$ ,  $\mathbb{P}(\Xi_2^c) \leq C(d)m^{-4}$ .*



*Proof.* The complement  $\Xi_2^c$  of  $\Xi_2$  is

$$\Xi_2^c := \{\exists 1 \leq |j| \leq M_m^\alpha : |[f]_j/\widehat{[f]}_j - 1| > 1/2 \text{ or } |\widehat{[f]}_j|^2 < 1/m\}.$$

Owing to statement **c)** from Lemma B.1 we have  $|[f]_j|^2 \geq 2/m$  for all  $j \in \{1, \dots, M_m^\alpha\}$ . In case that  $|\widehat{[f]}_j|^2 < 1/m$  a direct calculation using the reverse triangle inequality shows that  $|\widehat{[f]}_j/[f]_j - 1| \geq 1/\sqrt{2} - 1 > 1/4$ . In case that  $|[f]_j/\widehat{[f]}_j - 1| > \frac{1}{2}$ , one obtains  $|\widehat{[f]}_j/[f]_j - 1| > 1/3$ , and thus together we have

$$\Xi_2^c \subseteq \{\exists 1 \leq |j| \leq M_m^\alpha : |\widehat{[f]}_j/[f]_j - 1| > 1/4\}.$$

Now, Hoeffding's inequality implies

$$\mathbb{P}(|\widehat{[f]}_j/[f]_j - 1| > 1/4) \leq 4 \exp\left(-\frac{m|[f]_j|^2}{128}\right) \leq 4 \exp\left(-\frac{m\alpha_{M_m^\alpha}}{128d}\right),$$

and the statement of the lemma follows from statement **b)** of Lemma B.1 and the estimate  $M_m^\alpha \leq m$  which holds by definition of  $M_m^\alpha$ .  $\square$

LEMMA B.6 *Let Assumptions A and B hold. The event  $\Xi_3$  defined in (B.2) satisfies  $\mathbb{P}(\Xi_3^c) \leq C(\alpha, d)m^{-4}$  for all  $m \in \mathbb{N}$ .*

*Proof.* Let us consider the random sets

$$\Xi_{31} := \{N_n^{\alpha-} \wedge M_m^{\alpha-} > \widehat{K}_{nm}\} \quad \text{and} \quad \Xi_{32} := \{\widehat{K}_{nm} > N_n^{\alpha+} \wedge M_m^{\alpha+}\}.$$

Then,  $\Xi_3^c = \Xi_{31} \cup \Xi_{32}$  and we establish bounds for  $\mathbb{P}(\Xi_{31})$  and  $\mathbb{P}(\Xi_{32})$ , separately.

*Upper bound for  $\mathbb{P}(\Xi_{31})$ :* We use the equality  $\Xi_{31} = \{\widehat{N}_n < K_{nm}^{\alpha-}\} \cup \{\widehat{M}_m < K_{nm}^{\alpha-}\}$ . Owing to the definition of  $N_n^{\alpha-}$ , we have  $|[f]_j|^2 / ((2j+1)\omega_j^+) \geq 4 \log(n+4)/n$  for all  $j \in \{0, \dots, N_n^{\alpha-}\}$ , which yields

$$\begin{aligned} \{\widehat{N}_n < K_{nm}^{\alpha-}\} &\subseteq \{\exists 1 \leq j \leq K_{nm}^{\alpha-} : |\widehat{[f]}_j|^2 / ((2j+1)\omega_j^+) < \log(n+4)/n\} \\ &\subseteq \bigcup_{1 \leq j \leq K_{nm}^{\alpha-}} \{|\widehat{[f]}_j|/|[f]_j| \leq 1/2\} \\ &\subseteq \bigcup_{1 \leq j \leq K_{nm}^{\alpha-}} \{|\widehat{[f]}_j/[f]_j - 1| \geq 1/2\}. \end{aligned}$$

In a similar way, we obtain  $\{\widehat{M}_m < K_{nm}^{\alpha-}\} \subseteq \bigcup_{0 \leq j \leq K_{nm}^{\alpha-}} \{|\widehat{[f]}_j/[f]_j - 1| \geq 1/2\}$ . Thus, since  $M_m^{\alpha-} \leq M_m^{\alpha+}$  by definition, we have

$$\Xi_{31} \subseteq \bigcup_{1 \leq j \leq M_m^{\alpha+}} \{|\widehat{[f]}_j/[f]_j - 1| \geq 1/2\}.$$

Applying Hoeffding's inequality as in the proof of Lemma B.5 and exploiting Assumption B yields

$$\mathbb{P}(\Xi_{31}) \leq 4 \sum_{1 \leq |j| \leq M_m^{\alpha+}} \exp\left(-\frac{m|[f]_j|^2}{128}\right) \leq C(\alpha, d) \cdot m^{-4}. \quad (\text{B.13})$$

*Upper bound for  $\mathbb{P}(\Xi_{32})$ :* First, note that  $\Xi_{32} = \{\widehat{N}_n > K_{nm}^{\alpha+}\} \cap \{\widehat{M}_m > K_{nm}^{\alpha+}\}$ . In particular,  $K_{nm}^{\alpha+} < n \wedge m$ . If  $K_{nm}^{\alpha+} = N_n^{\alpha+} < n$ , we obtain

$$\Xi_{32} \subseteq \{\widehat{N}_n > N_n^{\alpha+}\} \subseteq \{\forall 1 \leq j \leq N_n^{\alpha+} + 1 : |\widehat{[f]}_j|^2 / ((2j+1)\omega_j^+) \geq \log(n+4)/n\}$$

$$\subseteq \{|\widehat{[f]}_{N_n^{\alpha+}+1}/|[f]_{N_n^{\alpha+}+1}| \geq 2\} \subseteq \{|\widehat{[f]}_{N_n^{\alpha+}+1}/|[f]_{N_n^{\alpha+}+1}| - 1| \geq 1\}.$$

Analogously, if  $K_{nm}^{\alpha+} = M_m^{\alpha+} < m$ , using  $m^{-1} \log m \geq 4|[f]_{M_m^{\alpha+}+1}|^2$  yields

$$\Xi_{32} \subseteq \{\widehat{M}_m > M_m^{\alpha+}\} \subseteq \{|\widehat{[f]}_{M_m^{\alpha+}+1}/|[f]_{M_m^{\alpha+}+1}| - 1| \geq 1\}$$

and thus  $\Xi_{32} \subseteq \{|\widehat{[f]}_{K_{nm}^{\alpha+}+1}/|[f]_{K_{nm}^{\alpha+}+1}| - 1| \geq 1\}$ . Application of Hoeffding's inequality and exploiting Assumption B yields

$$\mathbb{P}(\Xi_{32}) \leq 4 \exp\left(-\frac{m|[f]_{K_{nm}^{\alpha+}+1}|^2}{128}\right) \leq 4 \exp\left(-\frac{m\alpha_{M_m^{\alpha+}+1}}{128d}\right) \leq C(\alpha, d)m^{-5}. \quad (\text{B.14})$$

The statement of the lemma follows by combining Equations (B.13) and (B.14).  $\square$

## C. Proofs of Section 5

### C.1. Proof of Theorem 5.1

We define all the quantities appearing in this proof exactly as in the proof of Theorem 4.1 unless otherwise stated. We use the decomposition

$$\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2] = \square_1 + \square_2 + \square_3$$

established in the proof of Theorem 4.1 and use exactly the same arguments as in that proof to bound the terms  $\square_2$  and  $\square_3$ . Thus, it remains to find an appropriate bound for  $\square_1$ . In order to get such a bound, we first proceed as in the proof of Theorem 4.1 in order to obtain on  $\Xi_1 \cap \Xi_2$  the estimate

$$\|\widehat{\lambda}_k - \lambda\|_\omega^2 \leq 7r\omega_k\gamma_k^{-1} + 4\widetilde{\text{PEN}}_k - 4\widetilde{\text{PEN}}_{\widetilde{k}} + 40 \sup_{t \in \mathcal{B}_{k \vee \widetilde{k}}} |\langle \widetilde{\Theta}_{n \wedge m}, t \rangle_\omega|^2 + 32 \sup_{t \in \mathcal{B}_{k \vee \widetilde{k}}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2. \quad (\text{C.1})$$

Let us now introduce the quantity

$$\ddot{\lambda}_k := \sum_{0 \leq |j| \leq k} \frac{\mathbb{E}[\widehat{[\ell]}_j | \varepsilon]}{[f]_j} \mathbf{e}_j$$

where  $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n)$  is the vector containing the *unobserved* shifts  $\varepsilon_i$  in (2.5). Using the decomposition  $\widetilde{\Theta}_{n \wedge m} = \widetilde{\lambda}_{n \wedge m} - \lambda_{n \wedge m} = \widetilde{\lambda}_{n \wedge m} - \ddot{\lambda}_{n \wedge m} + \ddot{\lambda}_{n \wedge m} - \lambda_{n \wedge m}$  and setting

$$\Theta_{n \wedge m}^{(1)} := \widetilde{\lambda}_{n \wedge m} - \ddot{\lambda}_{n \wedge m} \quad \text{and} \quad \Theta_{n \wedge m}^{(2)} := \ddot{\lambda}_{n \wedge m} - \lambda_{n \wedge m}$$

we obtain from (C.1) that on  $\Xi_1 \cap \Xi_2$

$$\begin{aligned} \|\widehat{\lambda}_k - \lambda\|_\omega^2 &\leq 7r\omega_k\gamma_k^{-1} + 4\widetilde{\text{PEN}}_k - 4\widetilde{\text{PEN}}_{\widetilde{k}} + 80 \sup_{t \in \mathcal{B}_{k \vee \widetilde{k}}} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 \\ &\quad + 80 \sup_{t \in \mathcal{B}_{k \vee \widetilde{k}}} |\langle \Theta_{n \wedge m}^{(2)}, t \rangle_\omega|^2 + 32 \sup_{t \in \mathcal{B}_{k \vee \widetilde{k}}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2. \end{aligned}$$

Taking expectations, we obtain in analogy to the proof of Theorem 4.1 that

$$\mathbb{E}[\|\widehat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_1 \cap \Xi_2}] \leq C \min_{0 \leq k \leq K_{nm}^{\alpha+}} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\mathbb{D}_k^\alpha \log(n+2)}{n} \right\}$$

$$\begin{aligned}
& + 80 \sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0 \vee 1)}{n} \right)_+ \right] \\
& + 80 \sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(2)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0^2 \vee 1)}{n} \right)_+ \right] \\
& + 32 \mathbb{E} \left[ \sup_{t \in \mathcal{B}_{k \vee \tilde{k}}} |\langle \check{\Theta}_{n \wedge m}, t \rangle_\omega|^2 \right]. \tag{C.2}
\end{aligned}$$

We have

$$\begin{aligned}
& \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0 \vee 1)}{n} \right)_+ \right] \\
& = \mathbb{E} \left[ \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0 \vee 1)}{n} \right)_+ \mid \varepsilon \right] \right]
\end{aligned}$$

We apply Lemma C.2 with  $\delta_k^* = d\mathbb{D}_k^\alpha$  in order to obtain

$$\begin{aligned}
& \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0 \vee 1)}{n} \right)_+ \mid \varepsilon \right] \\
& \lesssim \frac{\mathbb{D}_k^\alpha}{n^3} + \frac{\mathbb{D}_k^\alpha}{n^2} \exp(-K_2 \sqrt{n \log(n+2)}).
\end{aligned}$$

Hence

$$\mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0 \vee 1)}{n} \right)_+ \right] \lesssim \frac{\mathbb{D}_k^\alpha}{n^3} + \frac{\mathbb{D}_k^\alpha}{n^2} \exp(-K_2 \sqrt{n \log(n+2)}).$$

By definition of  $\Xi$  we have  $K_{nm}^\alpha \leq N_n^\alpha$  and hence by the definition of  $N_n^\alpha$  that

$$\mathbb{D}_k^\alpha \leq \mathbb{D}_{N_n^\alpha}^\alpha = \sum_{0 \leq |j| \leq N_n^\alpha} \frac{\omega_j}{\alpha_j} \leq \frac{n}{\log(n+3)} \sum_{0 \leq |j| \leq N_n^\alpha} \frac{1}{2|j|+1} \lesssim n$$

where we obtain the last estimate thanks to the logarithmic increase of the harmonic series. Due to  $K_{nm}^\alpha \leq n$  we obtain

$$\sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0 \vee 1)}{n} \right)_+ \right] \lesssim \frac{1}{n}.$$

Applying Lemma C.4 with  $\delta_k^* = d\mathbb{D}_k^\alpha$  we obtain that

$$\begin{aligned}
& \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(2)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0^2 \vee 1)}{n} \right)_+ \right] \lesssim \frac{\mathbb{D}_k^\alpha}{n} \exp(-2 \log(n+2)) \\
& \quad + \frac{\mathbb{D}_k^\alpha}{n^2} \exp(-K_2 \sqrt{n \log(n)})
\end{aligned}$$

Using the relation  $\mathbb{D}_k^\alpha \lesssim n$  from above we obtain

$$\sum_{k=0}^{K_{nm}^\alpha} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(2)}, t \rangle_\omega|^2 - \frac{100 \log(n+2) d\mathbb{D}_k^\alpha([\ell]_0^2 \vee 1)}{n} \right)_+ \right] \lesssim \frac{1}{n}.$$

Finally, bounding the last term in (C.2) by means of Lemma B.3 we obtain from (C.2) using the obtained estimates that

$$\mathbb{E}[\|\hat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_{\Xi_1 \cap \Xi_2}] \leq \min_{0 \leq k \leq K_{nm}^\alpha} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\mathbb{D}_k^\alpha \log(n+2)}{n} \right\} + \Phi_m + \frac{1}{n}.$$

This shows the desired lower bound for  $\square_1$  and combining it with the bounds for  $\square_2$  and  $\square_3$  yields the result.  $\square$

### C.2. Proof of Theorem 5.2

We use the same decomposition as in the Proof of Theorem 4.2. In particular, all the quantities arising in the sequel are defined as in the proof of Theorem 4.2 unless otherwise stated. The terms  $\square_2$ ,  $\square_3$ , and  $\square_4$  are bounded exactly as in the proof of Theorem 4.2 and it remains find an appropriate bound for  $\square_1$ . Set  $\mathbb{D}_k := \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2}$  and

$$\text{PEN}_k = 2000\eta^{-1} \cdot (\widehat{[\ell]}_0 \vee 1) \cdot \frac{\mathbb{D}_k \log(n+2)}{n} + 2000\eta^{-2} \cdot (\widehat{[\ell]}_0^2 \vee 1) \cdot \frac{\mathbb{D}_k \log(n+2)}{n}.$$

From the definition of  $\text{PEN}_k$  and  $\widehat{\text{PEN}}_k$  one immediately obtains that on  $\Xi$

$$\text{PEN}_k \leq \widehat{\text{PEN}}_k \leq 9\text{PEN}_k$$

from which one follows that

$$(\text{PEN}_{k \vee \widehat{k}} + \widehat{\text{PEN}}_k - \widehat{\text{PEN}}_{\widehat{k}}) \mathbb{1}_\Xi \leq (\text{PEN}_k + \text{PEN}_{\widehat{k}} + \widehat{\text{PEN}}_k - \widehat{\text{PEN}}_{\widehat{k}}) \mathbb{1}_\Xi \leq 10\text{PEN}_k.$$

Now, combining the arguments from the proofs of Theorem 4.2 and 5.1 one can show

$$\sup_{\lambda \in \Lambda_\gamma^r} \sup_{f \in \mathcal{F}_\alpha^d} \mathbb{E}[\|\hat{\lambda}_k - \lambda\|_\omega^2 \mathbb{1}_\Xi] \lesssim \min_{0 \leq k \leq K_{nm}^\alpha} \max \left\{ \frac{\omega_k}{\gamma_k}, \frac{\mathbb{D}_k^\alpha \log(n+2)}{n} \right\} + \Phi_m + \frac{1}{n}.$$

The statement of the Theorem follows now by combining the bounds established for  $\square_1$ ,  $\square_2$ ,  $\square_3$ , and  $\square_4$ .  $\square$

### C.3. Auxiliary results

The following result is a conditional version of Proposition C.1 in [Kro16]. Since the proof is exactly the same as the one in the unconditional case (with Poisson processes instead of Cox processes) we omit its proof.

**PROPOSITION C.1** *Let  $N_1, \dots, N_n$  be Cox processes on a Polish space  $\mathbb{X}$  driven by finite random measures  $\eta_1, \dots, \eta_n$ , respectively. Set  $\nu_n(r) = \frac{1}{n} \sum_{k=1}^n \{ \int_{\mathbb{X}} r(x) dN_k(x) - \int_{\mathbb{X}} r(x) d\eta_k(x) \}$  for  $r$  contained in a countable class  $\mathcal{R}$  of complex-valued measurable functions. Denote  $\boldsymbol{\eta} = (\eta_1, \dots, \eta_n)$ . Then, for any  $\varepsilon > 0$ , there exist constants  $c_1, c_2 = \frac{1}{6}$ , and  $c_3$  such that*

$$\begin{aligned} & \mathbb{E} \left[ \left( \sup_{r \in \mathcal{R}} |\nu_n(r)|^2 - c(\varepsilon) H^2 \right)_+ \middle| \boldsymbol{\eta} \right] \\ & \leq c_1 \left\{ \frac{2v}{n} \exp \left( -c_2 \varepsilon \frac{nH^2}{v} \right) + \frac{M_1^2}{C^2(\varepsilon)n^2} \exp \left( -c_3 C(\varepsilon) \sqrt{\varepsilon} \frac{nH}{M_1} \right) \right\} \end{aligned}$$

where  $C(\varepsilon) = (\sqrt{1+\varepsilon} - 1) \wedge 1$ ,  $c(\varepsilon) = 4(1+2\varepsilon)$  and  $M_1$ ,  $H$  and  $v$  are such that

$$\sup_{r \in \mathcal{R}} \|r\|_\infty \leq M_1, \quad \mathbb{E} \left[ \sup_{r \in \mathcal{R}} |\nu_n(r)| \mid \boldsymbol{\eta} \right] \leq H, \quad \text{and} \quad \sup_{r \in \mathcal{R}} \text{Var} \left( \int_{\mathbb{X}} r(x) dN_k(x) \mid \boldsymbol{\eta} \right) \leq v \forall k.$$

LEMMA C.2 Let  $(\delta_k^*)_{k \in \mathbb{N}_0}$  be a sequence such that  $\delta_k^* \geq \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2}$  for all  $k \in \mathbb{N}_0$ . Then,

$$\begin{aligned} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle|^2 - \frac{100 \log(n+2) \delta_k^* ([\ell]_0 \vee 1)}{n} \right)_+ \mid \varepsilon \right] \\ \leq K_1 \left\{ \frac{\delta_k^* ([\ell]_0 \vee 1)}{n} \exp(-2 \log(n+2)) + \frac{\delta_k^*}{n^2} \exp \left( -K_2 \sqrt{n \log(n+2)} \right) \right\}, \end{aligned}$$

with strictly positive numerical constants  $K_1$  and  $K_2$ .

*Proof.* Putting  $r_t = \sum_{0 \leq |j| \leq k} \omega_j [f]_{-j}^{-1} \overline{[t]}_{-j} \mathbf{e}_j$ , it is easy to check that given  $\varepsilon$

$$\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega = \frac{1}{n} \sum_{i=1}^n \int_0^1 r_t(x) (dN_i(x) - \lambda_{\varepsilon_i}(x) dx)$$

where  $\lambda_\varepsilon(x) = \lambda(x - \varepsilon - \lfloor x - \varepsilon \rfloor)$ . Thus, we are in the framework of Proposition C.1 and it remains to find suitable constants  $M_1$ ,  $H$ , and  $v$  satisfying its preconditions.

*Condition concerning  $M_1$ :* We have

$$\sup_{t \in \mathcal{B}_k} \|r_t\|_\infty^2 = \sup_{t \in \mathcal{B}_k} \sup_{y \in [0,1)} |r_t(y)|^2 \leq \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2} \leq \delta_k^*$$

and one can choose  $M_1 = (\delta_k^*)^{\frac{1}{2}}$ .

*Condition concerning  $H$ :* We have

$$\mathbb{E} \left[ \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(1)}, t \rangle_\omega|^2 \mid \varepsilon \right] = \frac{[\ell]_0}{n} \cdot \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2} \leq \frac{([\ell]_0 \vee 1) \delta_k^* \log(n+2)}{n}$$

and one can choose  $H = \left( \frac{([\ell]_0 \vee 1) \delta_k^* \log(n+2)}{n} \right)^{1/2}$ .

*Condition concerning  $v$ :* It holds that

$$\text{Var} \left( \int_0^1 r_t(x) N_k(x) \mid \varepsilon \right) = \int_0^1 |r_t(x)|^2 \lambda_{\varepsilon_k}(x) dx \leq \left( \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2} \right) \cdot [\ell]_0 \leq \delta_k^* \cdot ([\ell]_0 \vee 1)$$

and one can choose  $v = \delta_k^* \cdot ([\ell]_0 \vee 1)$ . The statement of the Lemma follows now by applying Proposition C.1 with  $\varepsilon = 12$ .  $\square$

PROPOSITION C.3 Let  $X_1, \dots, X_n$  random variables with values in some Polish space. Let  $\nu_n(r) = \frac{1}{n} \sum_{i=1}^n r(X_i) - \mathbb{E}[r(X_i)]$  for  $r$  belonging to some countable set  $\mathcal{R}$  of measurable complex-valued functions. Then, for all  $\varepsilon > 0$ , there are constants  $c_1$ ,  $c_2 = \frac{1}{6}$  and  $c_3$  such that

$$\mathbb{E} \left[ \left( \sup_{r \in \mathcal{R}} |\nu_n(r)|^2 - c(\varepsilon) H^2 \right)_+ \right] \leq c_1 \left\{ \frac{2v}{n} \exp \left( -c_2 \varepsilon \frac{n H^2}{v} \right) + \frac{M_1^2}{C^2(\varepsilon) n^2} \exp \left( -c_3 C(\varepsilon) \sqrt{\varepsilon} \frac{n H}{M_1} \right) \right\},$$

with  $C(\varepsilon) = (\sqrt{1+\varepsilon} - 1) \wedge 1$ ,  $c(\varepsilon) = 4(1 + 2\varepsilon)$  and

$$\sup_{r \in \mathcal{R}} \|r\|_\infty \leq M_1, \quad \mathbb{E} \left[ \sup_{r \in \mathcal{R}} |\nu_n(r)| \right] \leq H, \quad \sup_{r \in \mathcal{R}} \text{Var}(r(X_k)) \leq v \forall k.$$

*Proof.* The proof consists of an easy adaption of the proof given in [Cha13] to the complex-valued case and the statement follows by bookkeeping of the occurring numerical constants.  $\square$

LEMMA C.4 Let  $(\delta_k^*)_{k \in \mathbb{N}_0}$  be a sequence such that  $\delta_k^* \geq \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2}$  for all  $k \in \mathbb{N}_0$ . Then

$$\begin{aligned} \mathbb{E} \left[ \left( \sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(2)}, t \rangle|^2 - \frac{100 \log(n+2) \delta_k^* ([\ell]_0^2 \vee 1)}{n} \right)_+ \right] \\ \leq K_1 \left\{ \frac{\delta_k^* ([\ell]_0^2 \vee 1)}{n^3} + \frac{([\ell]_0^2 \vee 1) \delta_k^*}{n^2} \cdot \exp(-K_2 \sqrt{n \log(n+2)}) \right\}. \end{aligned}$$

*Proof.* We define  $r'_t(x) = \sum_{0 \leq |j| \leq k} \omega_j [f]_{-j}^{-1} [\overline{t}]_{-j} \mathbf{e}_j$  which coincides with the definition of  $r_t$  in the proof of Lemma C.2. Then, we have

$$\langle \Theta_{n \wedge m}^{(2)}, t \rangle_\omega = \frac{1}{n} \sum_{i=1}^n \int_0^1 r'_t(x) \lambda_{\varepsilon_i}(x) dx - \int_0^1 r'_t(x) \ell(x) dx.$$

Setting  $r_t(\varepsilon_i) = \int_0^1 r'_t(x) \lambda_{\varepsilon_i}(x) dx$ , we are in the framework of Proposition C.3 and it remains to find suitable constants  $M_1$ ,  $H$  and  $v$  satisfying the preconditions of this proposition.

*Condition concerning  $M_1$ :* Note that the definition of  $r'_t$  is the same as the definition of  $r_t$  in the proof of Lemma C.2. Thus we obtain

$$\sup_{t \in \mathcal{B}_k} \|r_t\|_\infty = \sup_{\varepsilon \in [0,1]} \sup_{t \in \mathcal{B}_k} \left| \int_0^1 r'_t(x) \lambda_\varepsilon(x) dx \right| \leq (\delta_k^*)^{1/2} \cdot \sup_{\varepsilon \in [0,1]} \int_0^1 \lambda_\varepsilon(x) dx = (\delta_k^*)^{1/2} \cdot ([\ell]_0 \vee 1)$$

and we can take  $M_1 = (\delta_k^*)^{1/2} \cdot ([\ell]_0 \vee 1)$ .

*Condition concerning  $H$ :* We have

$$\begin{aligned} \mathbb{E} [\sup_{t \in \mathcal{B}_k} |\langle \Theta_{n \wedge m}^{(2)}, t \rangle_\omega|^2] &\leq \left( \sum_{0 \leq |j| \leq k} \frac{\omega_j}{|[f]_j|^2} \right) \frac{1}{n} \mathbb{E} [|\int_0^1 \mathbf{e}_j(x) (\lambda_{\varepsilon_1}(x) dx - \ell(x) dx)|^2] \\ &\leq \frac{\delta_k^* [\ell]_0^2}{n} \leq \frac{\delta_k^* [\ell]_0^2 \log(n+2)}{n} \end{aligned}$$

and we can set  $H = \left( \frac{\delta_k^* \log(n+2)}{n} \right)^{1/2} \cdot ([\ell]_0 \vee 1)$ .

*Condition concerning  $v$ :* It holds

$$\text{Var}(r_t(\varepsilon_i)) \leq \mathbb{E} \left[ \left| \int_0^1 r'_t(x) \lambda_{\varepsilon_i}(x) dx \right|^2 \right] \leq [\ell]_0^2 \cdot \mathbb{E} \left[ \int_0^1 |r'_t(x)|^2 \frac{\lambda_{\varepsilon_i}(x)}{[\lambda]_0} dx \right] \leq ([\ell]_0^2 \vee 1) \cdot \delta_k^*$$

and we define  $v = ([\ell]_0^2 \vee 1) \cdot \delta_k^*$ . Now that statement of the lemma follows from Proposition C.3 with  $\varepsilon = 12$ .  $\square$

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